Modeling, Verification and Control of complex Systems: From foundations to power network applications

Report D4.3
Final report on modeling, analysis, impact and potential exploitation
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1 Introduction

1.1 MoVeS Summary

The aim is to propose novel methods for modelling, analysis and control of complex, large scale systems. Fundamental research is motivated by applied problems in power networks. We adopt the framework of stochastic hybrid systems (SHS), which allows one to capture the interaction between continuous dynamics, discrete dynamics and probabilistic uncertainty. In the context of power networks, SHS arise naturally: continuous dynamics model the evolution of voltages, frequencies, etc., discrete dynamics model changes in network topology, and probability models the uncertainty about power demand and (with the advent of renewables) power supply. More generally, because of their versatility, SHS are recognized as an ideal framework for capturing the intricacies of complex, large scale systems. Motivated by this, considerable research effort has been devoted to the development of modelling, analysis and control methods for SHS, in computer science (giving rise to theorem proving and model checking methods) and in control engineering (giving rise to optimal control and randomized methods). Despite several success stories, however, none of the methods currently available are powerful enough to deal with real life large scale applications. We feel that a key reason for this is that the methods have been developed by different communities in relative isolation, motivated by different applications. As a consequence synergies between them have never been fully explored. We propose to systematically exploit such synergies. Our multi-disciplinary team, which brings together experts on all the state of the art SHS methods, will establish links between model checking, theorem proving, optimal control and randomized methods. Leveraging on their complementary strengths we will develop combined strategies and tools to enable novel applications to complex, large scale systems. Common power networks case studies will provide a testing ground for the fundamental developments, motivate them, and keep them focused.

1.2 Objectives of WP4

The aim of the work package is to demonstrate the application and potential impact of novel methods for the modelling, analysis and control of stochastic hybrid systems developed in the project. This was accomplished through selected case studies from the area of power networks, which provide an ideal testing ground for stochastic hybrid systems, since by nature they involve a tight coupling of continuous dynamics (governing, for example, the power flows and the fluctuations of voltages, frequencies, etc.), discrete dynamics (for example, the positions of switches, protection and isolation devices, trans-
former taps, etc.), and uncertainty (in the demand for power, but also, with the recent emphasis on renewable energy and distributed generation, in its supply).

The uncertainties mentioned above give rise both to reliability concerns and to price fluctuations in the power markets and this coupling was also taken into account. In this work package we explore these issues by developing a series of case studies and by applying the fundamental theoretical advances investigated in the project. The work was organized around the following tasks.

**Task 4.0:**

Methodological integration. The aim of this task was to highlight synergies in the methodological developments related to the theoretical work.

**Task 4.1:**

Identification of promising case studies. The work started with a wide sweep for promising problems. The aim was to establish problems that are both important from the point of view of power systems and for which stochastic uncertainty appears to be prominent. In the process the definition of the areas (reliability and pricing) was refined, the boundaries between them were clarified, and other areas in power networks where stochastic uncertainty may play an important role identified. The results were documented in the deliverable D4.1.

**Task 4.2:**

Case study selection. This task indicated which of the problems identified under Task 4.1 ought to be pursued further as MoVeS case studies. The main goal was to select the problems for which the stochastic hybrid aspects are the most prominent. We narrowed the field down to three case studies, best suited for our needs. The criteria for the selection included the potential impact for power networks and the potential for demonstrating the methods developed in the project. The results were documented in the deliverable D4.1: the three main case studies were identified as “microgrid energy management”, “real-time demand response” and “reserve markets and ancillary services”.

**Task 4.3:**

Development of stochastic hybrid models. The methods developed under WP1 were applied to the case studies. In this task we developed quantitative stochastic hybrid models for each case study. These detailed and realistic models were further used as testbeds for assessing the performance of the methods developed under Task 4.4. Through a process of abstraction, simpler high-level models were obtained, and subsequently customized to enable the application of the analysis and control methods under Task 4.4.
Task 4.4:
Application of analysis and design methods. The methods developed in WP2 and WP3 were applied to the models developed under Task 4.3. An effort was made to apply as wide range of results as possible by exploring different facets of the three case studies. This helped to guide the development of the simplified models under Task 4.3. The results of Task 4.4, together with those of Task 4.3, formed the basis of the deliverable D4.2.

Task 4.5:
Impact analysis and exploitation. The objective of this task was to explore the implications of the previous results for power systems. One aspect of this related to the use of detailed simulation models developed under Task 4.3 to exhaustively investigate the performance of the methods under realistic conditions. The other was oriented towards assessment of the financial, security, and regulatory impacts of the results.

1.3 Objectives of D4.3
The deliverable D4.3 is the final report on modeling, analysis, impact and potential exploitation. The report summarizes the final conclusions of Task 4.4 and Task 4.5. It also outlines potential avenues for follow-up exploitation.

<table>
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Table 1.1: Application of developed methods on the case studies

1.4 Document Structure
In Chapter 2, the methods developed in individual work packages were applied to the selected case study problems identified previously in the deliverable D4.1. The Table captures the mapping between methods and selected case studies.
Composition and abstraction methods developed under work package WP1 were applied to the real time demand response and microgrid case study. Methods from work package WP2 – Satisfiability modulo theory and model checking methods – were applied to the same case studies as methods from WP1. Microgrids and Reserve scheduling serve as test cases for approximate dynamic programming methods developed under WP3. Finally stochastic MPC methods were applied to Reserve Scheduling case study.

The impact and potential exploitation for each case study is discussed in the second part of this document.
Chapter summarizes the application of the methods developed within MoVeS project on the selected case studies. Chapter is divided into several sections based on the related case study—Microgrid Energy Management, Real Time Demand Response, and N-1 Security and Reserve Scheduling.

2.1 Microgrid Energy Management

This section focuses on the application of developed methods on the microgrid case study. The section is divided into several parts which relate to the particular work package of the MoVeS project. The relationship of the particular subsections is depicted on the Figure 2.1.

First section 2.1.1 Microgrid Overview briefly describes the considered configuration of the microgrid and its components. The models of the microgrid emerged from the activities related to WP4 and more detailed description of particular elements of the microgrid can be found in the Deliverable D4.2 [6]. Section 2.1.2 Modelling and Statistical Model Checking of a Microgrid focuses on analysis of the microgrid based on the logic PCTL, using the statistical model checker Uppaal-SMC which corresponds with composition (WP1) and model checking methods.
(WP2). Following sections focuses on the application of approximate dynamic programming (WP3) on the microgrid energy management problem. Section 2.1.3 ADP based on Q Iteration describes use of ADP based Q iteration technique while the section 2.1.4 ADP based on policy search shows the application of the policy search approach.

2.1.1 Microgrid Overview

Microgrids (MGs) are small-scale energy networks that supply electricity, heating and cooling for a small community (e.g. academic campus etc.) [35]. MG can include various energy generation, consumption, storage, distribution and transfer devices. MG energy management system ensures proper control of particular devices and their coordination which is important for stable and economically efficient operation of a MG. In this section the model of considered microgrid is briefly described. Considered configuration of the microgrid is depicted on the Figure 2.2. Depicted scheme stands for full configuration of the microgrid which consists of elements of various type – generation, transfer, transformation, consumption and storage elements. Detailed description of particular elements of the Microgrid can be found in the deliverable D4.2 [6]. Here the brief overview of the elements will be provided.

![Figure 2.2: Scheme of considered microgrid](image)

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Local Power Network

The Local Power Network constitutes the junction point that interconnects the main distribution grid, local generators, electrical storage and power loads. The power balance equation for the LPN is given by:

\[ P_G(t) + P_W(t) + P_M(t) + P_E(t) = \sum_{i=1}^{N} P_{Ch,i}(t) + P_L(t), \]  

(2.1)

where \( P_G \) is the power load from the grid, \( P_W \) is the power generated by the wind turbine, \( P_M \) stands for the power generated by the microturbine, \( P_E \) denotes the power load/supply to/from electrical storage, \( P_L \) is the electrical load and \( P_{Ch,i} \) is the power demand of the \( i \)-th chiller. In the document, we do not consider the grid inertia (grid dynamics) because it affects phenomena at much smaller time scale than the ones studied here.

The stability of the LPN depends on the balance between power supply (left-hand side of Eq. (2.1)) and demand (right-hand side of Eq. (2.1)). The power imbalance \( \Delta P(t) \) denotes their difference. The frequency of the AC current \( f \) can be used as the stability criterion and can be considered as a function of power imbalance \( f(\Delta P(t)) \). For a stable operation of the microgrid, the frequency may vary only within a certain a priori-defined interval and for a specified amount of time. Large power imbalances may result in black-outs.

Renewable Power

Renewable power sources are represented by the wind turbine which is modelled by a nonlinear static model broadly spread in the literature (e.g. [63]). The model has following form

\[ P_W(t) = \frac{\pi}{2} \cdot \rho \cdot R^2 \cdot v_w^3(t) \cdot C_p(\eta(t), \theta(t)), \]  

(2.2)

where \( \rho \) denotes the air density, \( R \) is the blade radius, \( u \) denotes the wind speed, and \( C_p \) is an efficiency function depending on the wind direction \( \theta(t) \) and on the tip speed ratio \( \eta(t) = \frac{\omega R}{v_w(t)} \), where \( \omega \) is the speed of the blade tip. We consider the wind turbine to be connected to the LPN and to be operating in the power optimization area\(^2\). The static part of the model is affected by two stochastic inputs - the wind direction \( \theta \) and the wind speed \( u \). When we consider optimal placement of the wind turbine, then generated power \( P_W \) depends on the wind speed \( v_w \) only. For the modeling of the wind speed \( u \), we use first order Markov chain model as described in [34]. In our studies, we use a second-order polynomial approximation of this equation which is common information available from manufacturers’ datasheets (e.g. [7]).

\(^1\)Such as chilled water circuit temperature development.
\(^2\)The turbine is in the power optimization area when the speed of its blade tip is less than a maximal tip speed.
Grid Power

Grid power $P_G$ is power downloaded from main distribution grid and it stands for stable power source which has unconstrained capacity. If the microgrid is connected to the main distribution grid then the power $P_G$ eliminates all power imbalances in the LPN.

Electrical Energy Storage

Electrical storage is the device that enables storing electrical energy over time. We use the storage model of [85] where the electrical storage evolves according to the following differential equation:

$$\frac{dP_S}{dt} = -\gamma \cdot P_E(t) - P_{LOSS}(t), \quad (2.3)$$

where $P_S$ expresses the stored energy level in the storage, $\gamma$ denotes the power exchange efficiency, $P_E$ is the power exchanged between the storage and the LPN, and $P_{LOSS}$ stands for power losses associated to the storage.

Combined Heat and Power

The microturbine constitutes the combined source of heat and power which influences the electrical and thermal power dynamics of the microgrid. The operation of a microturbine can be divided into eight separate discrete modes [99]. Mode transitions are guarded by the turbine angular velocity $\omega$ whose evolution is defined as:

$$J \cdot \dot{\omega} = T_M(t) + T_E(t) - F_V(t),$$

where $J$ is the moment of inertia of the turbine, $T_E$ refers to the electrical torque, $T_M$ is the mechanical torque, and $F_V$ is the viscous friction defined by $F_V(t) = k_V \cdot \omega(t)$. Particular discrete modes are:

- **Off** – Device is turned off.
- **Start up** – Turbine accelerates to 25000 rpm ($\omega_{\text{start}}$). In this mode, the turbine generator acts as motor and consumes electrical power. The electrical power $P_M$ is the driving force of the turbine ($T_E$ is positive and $P_M$ is negative) and yields $T_E(t) = a_{0,M} + a_{1,M} \cdot P_M(t) + a_{2,M} \cdot P_M(t)^2$. The power requirement is modelled by a feedback loop in order to reach the desired angular velocity of $\omega_{\text{start}}$ in the following way:

$$P_M(t) = k_{P,M,P} \cdot (\omega_{\text{start}} - \omega(t)) + k_{P,M,I} \cdot \int_0^t (\omega_{\text{start}} - \omega(\tau)) \cdot d\tau.$$

- **Stabilization** – Short period before gas ignition where the angular speed of the turbine is constant $\omega = 25000$ rpm.
• **Acceleration** – Electrical torque from the generator drives the turbine to speed 45000 rpm. At the end of the interval, it comes to the ignition. The minimal fuel flow maintains the combustion process but does not produce mechanical torque. Torque $T_M$ is taken to be zero.

• **Warm up** – The microturbine consumes only fuel, whence $P_M = 0$. The energy from the combustion keeps the turbine at the desired speed of 45000 rpm. $T_M$ provides the necessary mechanical torque.

• **Operational** – The microturbine produces power and the turbine is driven of the fuel combustion. $T_M$ is the driving force for the rotation of turbine. Electrical power is generated ($P_M$ is positive) and the emerged electromagnetic field tends to slow down the turbine rotation hence ($T_E$ is negative):

\[
T_M(t) = a_{3,M} + a_{4,M} \cdot m_{f,CHP}(t) + a_{5,M} \cdot m_{f,CHP}(t)^2 - a_{6,M} \cdot \omega(t),
\]
\[
T_E(t) = a_{7,M} + a_{8,M} \cdot \omega(t) + a_{9,M} \cdot \omega(t)^2,
\]
\[
P_M(t) = a_{10,M} + a_{11,M} \cdot \omega(t) + a_{12,M} \cdot \omega(t)^2.
\]

The power at which the microturbine should eventually operate determines the desired speed ($\omega_{SP}$) of the turbine. The fuel flow $m_{f,CHP}$ is controlled by a PI controller to keep the turbine at a $\omega_{SP}$ according to:

\[
flow(t) = k_{m,f,P} \cdot (\omega_{SP} - \omega(t)) + k_{m,f,I} \cdot \int_0^t (\omega_{SP} - \omega(\tau))d\tau,
\]
\[
m_{f,CHP}(t) = \min\{m_{f,CHP}^{\text{min}}, flow(t)\}.
\]

• **Slow down** – Generator is disconnected from the LPN and the turbine is slowed down to $\omega = 25000$ rpm.

• **Cool down** – Fuel supply is closed ($m_{f,CHP} = 0$). Microturbine slows to stop ($\omega = 0$).

Heat produced from the combustion of fuel serves the heating load (e.g., domestic hot water) by:

\[
Q_M(t) = a_{13,M} + a_{14,M} \cdot m_{f,CHP}(t) + a_{15,M} \cdot m_{f,CHP}(t)^2.
\]

**Chiller**

Chillers are electrical devices that remove heat from a liquid via vapor compression or absorption cycle. We use the static nonlinear Gordon-Nq model [93] for modeling of the chiller behavior, whose model has following form:

\[
\frac{T_{ei}}{T_{ci}} \cdot \left(1 + \frac{1}{COP}\right) - 1 = \frac{T_{ei}}{Q_e} \cdot \Delta S_T + Q_{\text{leak,eqv}} \cdot \frac{(T_{ei} - T_{ci})}{T_{ei} \times Q_e} + \frac{R \times Q_e}{T_{ei}} \cdot \left(1 + \frac{1}{COP}\right),
\]

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where $T_{ei}$ is evaporator inlet water temperature, $T_{ci}$ denotes condenser inlet water temperature, $COP$ is coefficient of performance, $Q_e$ marks evaporator duty, $\Delta S_T$ total internal entropy, $Q_{\text{leak,eqv}}$ means equivalent heat leak and $R$ is heat exchanger thermal resistance. Coefficient of performance ($COP$) is given by ratio 

$$COP_i = \frac{Q_{Ch,i}}{P_{Ch,i}}$$

where $Q_{Ch,i}$ is evaporator duty, $P_{Ch,i}$ is compressor power of $i$-th chiller. We consider air cooled chiller where $T_{ci}$ constitutes outside temperature $T_{OA}$ and $T_{ei}$ can be considered as chilled water temperature $T_{CW}$. By substituting coefficient of performance $COP_i = \frac{Q_{Ch,i}}{P_{Ch,i}}$ into chiller equation 2.4, compressor power $P_{Ch,i}$ can be derived as follows:

$$P_{Ch,i}(t) = \frac{a_{1,Ch}^{(i)} \cdot T_{CW}(t) \cdot T_{OA}(t) + a_{2,Ch}^{(i)} \cdot (T_{OA}(t) - T_{CW}(t))}{T_{CW}(t) - a_{3,Ch}^{(i)} \cdot Q_{Ch,i}(t)}$$

$$+ \frac{a_{4,Ch}^{(i)} \cdot T_{OA}(t) \cdot Q_{Ch,i}(t)}{T_{CW}(t) - a_{3,Ch}^{(i)} \cdot Q_{Ch,i}(t)} - Q_{Ch,i}(t)$$

(2.5)

where $a_{1,Ch}^{(i)} = \Delta S_T$, $a_{2,Ch}^{(i)} = Q_{\text{leak}}$ and $a_{3,Ch}^{(i)} = R$ are coefficients of $i$-th chiller estimated from measured data or given by a table. The parameter $a_{4,Ch}^{(i)}$ stands for bias.

Power and Thermal Load

Dynamics of some loads can be captured by stochastic differential equation. Here we define dynamics of the electrical load which stands for unspecific electrical load (e.g., building equipment, computers, electrical heaters etc.). The electrical load can be modeled by the Uhlenbeck-Ornstein model [108] as follows:

$$dP_L = \alpha_L \cdot (\hat{P}_L(t) - P_L(t)) \cdot dt + \sigma_L \cdot dW,$$

(2.6)

where $\hat{P}_L$ is a given load profile, $\alpha_L$ represents a tracking coefficient, $\sigma_L$ is a variation coefficient, and $dW$ denotes the Wiener process. Load profile $P_L$ can be modeled or given by external forecaster. Here, we consider that the load profile forecast for the considered time period is available. The heating load represents a small demand for heating (e.g., domestic hot water) which is supplied by wasted heat from the microturbine. This load is modelled in a similar fashion as the electrical load above (Eq. 2.6).

Cooling Load

Cooling load can be modelled in various ways. In the document, we consider black box and physically based approaches.

a) **Black box approach** – cooling load is defined by stochastic differential equation which has similar form to the electrical load (Eq. 2.6).

b) **Physically based approach** – The cooling load is considered as the cooling load of a building consisting from several rooms (generally zones). The zone temperature
$T_{ZA}$ (at time $t$) evolves according to the stochastic differential equation:

$$C_{ZA} \frac{dT_{ZA}}{dt} = X_C(t) \cdot k_{cw} \cdot (T_{CW}(t) - T_{ZA}(t)) + k_{out} \cdot (T_{OA}(t) - T_{ZA}(t)) + NP(t) \cdot gain(T_{ZA}),$$

where $T_{CW}(t)$ is the temperature of the coolant, $X_{cool}(t)$ the thermostat valve position, $C_{ZA}$ is the thermal capacity of the zone, $k_{cw}$ is a heat transfer coefficient for zone-chilled water, $k_{out}$ is a heat transfer coefficient for zone-outside air and $gain(T_{ZA})$ stands for the heat produced by one person [86]. It is assumed that each person produces same amount of energy based on the function $gain(T_{ZA})$. The stochastic aspects here are the number of people $NP(t)$ in a zone and the outside air temperature $T_{OA}(t)$ which evolves based on SDE Eq. (2.13).

The zone is equipped by thermostat which function is to regulate the temperature of the room $T_{ZA}$ by changing the value of $X_C$. The thermostat is modelled by a PI controller and the value of $X_C$ depends on two other quantities, the desired temperature of the zone $T_{ZASP}$ and the current zone temperature $T_{ZA}$, along with proportional $k_{X_C,P}$ and integral $k_{X_C,I}$ constants:

$$e_C(t) = T_{ZA}(t) - T_{ZASP}(t), \text{ and}$$

$$temp(t) = k_{X_C,P} \cdot e_C(t) + k_{X_C,I} \cdot \int_0^t e_C(t) \, dt$$

$$X_C(t) = \max\{0, \min\{temp(t), 100\}\}.$$ (2.8)

### Chilled Water Circuit

Chilled water circuit (CHWC) interconnects thermal energy supplies with thermal loads. Under consideration of thermal dynamics, the CHWC can be modeled as follows:

$$C_{CW} \cdot T_{CW} = Q_C(t) - Q_S(t) - \sum_{i=1}^N Q_{Ch,i}(t),$$

where $C_{CW}$ is water circuit thermal capacity, $T_{CW}$ is chilled water temperature, $Q_C$ stands for the thermal load, $Q_S$ is thermal energy loaded (stored) from (to) thermal energy storage and $Q_{Ch,i}$ is energy production of $i$-th chiller. Temperature of the chilled water temperature is considered as $T_{CW} = 10^\circ$C.

### Thermal Storage

Thermal energy storage can store the energy over time. Although more complex and sophisticated storage models have been developed (e.g. [79]), we adopt a simplified storage model inspired by [85], which expresses the stored energy $Q_{ES}$ as

$$\dot{Q}_{ES}(t) = -\eta \cdot Q_E(t) - Q_{LOSS}(t),$$

where $\eta$ denotes energy exchange efficiency, $Q_E$ is the energy exchanged between the storage and the CHWC, and $Q_{LOSS}$ denotes thermal losses associated to the storage.
Note that the variable $Q_{ES}$ is the state variable determining the energy level stored in the storage device, whereas the variable $Q_E$ denotes an input variable quantifying the energy supplied (loaded) from (over) the storage device in connection with the CHWC. Thermal energy exchange efficiency parameter is defined as

$$
\eta = \begin{cases} 
\eta_s & \text{for the Supply mode } (Q_E > 0), \\
0 & \text{for the Store mode } (Q_E = 0), \\
\eta_l & \text{for the Load mode } (Q_E < 0).
\end{cases}
$$

(2.11)

**Boiler**

Boiler is a device which consumes gas $m_{fB}$ and produces heat energy $Q_B$. The boiler model is not described here because we considered small heating loads (e.g. domestic hot water) in our experiments, which are fully covered by waste heat from the microturbine. Details about boiler’s modelling can be found for example in [9].

**Hot Water Circuit**

Hot water circuit (HWC) interconnects heat energy supplies with heat loads. Similar as for CHWC, the HWC can be modeled as follows

$$
C_{HW} \cdot \frac{dT_{HW}}{dt} = Q_{CHP}(t) + Q_B(t) - Q_H(t),
$$

(2.12)

where $C_{HW}$ is water circuit thermal capacity, $T_{HW}$ is hot water temperature, $Q_{HL}$ stands for the heating load, $Q_{CHP}$ is thermal energy supplied by microturbine and $Q_B$ is heat delivered by boiler. Temperature of the hot water temperature is considered as $T_{HW} = 50^\circ C$.

**Outside Temperature**

Evolution of outside temperature has to be taken into account because it affects the performance of the air conditioned chillers. The outside air temperature $T_{OA}$ can evolve according to modified electrical load model

$$
dT_{OA} = \alpha_{OA} \cdot (\hat{T}_{OA}(t) - T_{OA}(t)) \cdot dt + \sigma_{OA} \cdot dW,
$$

(2.13)

where $\hat{T}_{OA}$ is forecasted outside temperature, $\alpha_{OA}$ stands for a tracking coefficient, $\sigma_{OA}$ is a variation coefficient, and $dW$ denotes the Wiener process.

**2.1.2 Modelling and Statistical Model Checking of a Microgrid**

This section reports on the modelling and analysis of a microgrid with wind, microturbines, and the main grid as generation resources. The microgrid is modelled as a parallel composition of various stochastic hybrid automata. Extensive simulation runs of the behaviour of the main individual microgrid components give insight into the complex dynamics of the system and provide useful information to determine adequate parameter settings. The analysis of the microgrid focuses on checking several temporal properties, expressed in the logic PCTL, using the statistical model checker UPPAAL-SMC.

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Introduction. We consider the microgrid configuration in Fig. 2.3 which is derived from the configuration described in the section 2.1.1. This microgrid is connected to the electricity and natural gas distribution grids and incorporates two local energy sources – wind turbine and a microturbine. The microturbine represents a deterministic energy source which produces electricity and heat. The heat can be utilized for satisfying the heating load such as domestic hot water. The wind turbine is of stochastic nature and its power output depends on actual wind properties. The local power network (LPN) stands for the junction point where the electricity supply meets the electricity demand e.g. chillers, electrical load, etc. Chillers remove the heat from the chilled water circuit (CHWC) which supplies the cooling load with the chill. Demand of cooling load as well as performances of chillers can be affected by the outside temperature. Electrical energy storage is connected to the LPN and enables the possibility to store/load electricity energy. This can be very useful for the optimization of operational costs of the microgrid.

We model this system as a composition of stochastic hybrid automata, and describe their continuous dynamics. For several components we have adopted the dynamics from the literature [43, 85, 99]. Extensive simulation runs of the behaviour of the microgrid components give insight into the complex dynamics of the system and provide useful information to determine adequate parameter settings. The parameter settings used in our verification models have been obtained in this way and have been validated by Honeywell. For the analysis of the microgrid, we resort to model checking. As the stochastic dynamics and complexity of the microgrid configuration go beyond the scope of the above techniques that typically rely on discretization and dynamic programming, we resort here to statistical model checking. This technique is basically employing Monte Carlo simulation in order to check the validity of temporal logic properties within a
given a priori-defined confidence interval. In order to analyse probabilistic reachability and invariance properties of the microgrid case study, we use UPPAAL-SMC, a recent extension to the model checker Uppaal (www.uppaal.org). UPPAAL-SMC originally focused on (priced and probabilistic) timed automata; however, recently it also covers networks of stochastic hybrid automata [36]. We report on the statistical model checking of several (bounded) PCTL formulas and investigate the run-times for various confidence intervals and time horizons.

As described above, we have modelled all individual components as stochastic hybrid automata. Let LPN, CH1, CH2, CHWC, Zi, MT, WT, ST, EL be the SHA models of the local power network, the two chillers, the chilled-water circuit, the i\(^{th}\) room (or zone), microturbine, wind turbine, storage, and electrical load, respectively. The composite model of the microgrid is now given as:

\[
MG = (LPN \parallel CH_1 \parallel CH_2 \parallel CHWC \parallel Z \parallel \ldots \parallel Z \parallel MT \parallel WT \parallel ST \parallel EL)
\]

where \(\parallel\) denotes a parallel composition operator.

**Simulation Results.** There are various input parameters for the different components of the microgrid. Controllers are designed to modify these inputs in order to satisfy some optimality criteria, usually to minimize power consumption. Various controllers can be modelled by automata. The effect of these controllers can be studied by running the respective automaton in parallel with the microgrid. For example, we can consider an automaton that depending on the required cooling power and ambient temperature, distributes the energy among the two chillers accordingly. It is possible that some chillers work better at lower energy requirements than others. The control parameter \(\alpha_{ch}(t)\) can redistribute the cooling energy production between chillers. In the following experiment (see Fig. 2.4), we can see that the total power consumed by the chiller is higher when a fixed \(\alpha\) is used in comparison to a more dynamic \(\alpha\) which depends on the cooling energy requirements.

We are also interested in testing the behaviour of the microgrid under special conditions, e.g. observing the frequency deviation over time when the microgrid is disconnected from the main grid (islander mode). Figure 2.5 shows the frequency deviation when the microgrid is running in islander mode with six wind turbines providing for two chillers and one electric load along with a storage device. Initially, the power generated by wind turbines is insufficient – in this experiment, it took some time for the wind turbine to pick up speed, so to speak – and hence the batteries were catering for the power deficit. At some point, the batteries run out and we have negative frequency deviation. When the six turbines accelerate, they provide (more than) enough energy for the demand yielding the positive deviation. Finally, the batteries start to recharge themselves to use the excess power and the frequency stabilizes.

**The UPPAAL-SMC Tool.** Numerical methods to solve model-checking problems of stochastic hybrid systems against some temporal formula are algorithmically involved
and suffer from the curse of dimensionality \[2, 3, 89\]. The main reason for this is the use of discretization. In contrast, statistical model checking avoids these problems by resorting to discrete-event simulation. In a nutshell, it generates and examines finitely many simulation runs of the model at hand, and uses hypothesis testing to infer whether the obtained simulation samples provide statistical evidence for the satisfaction or violation of some (temporal logic) specification \[107\]. We analyze the microgrid case study using the Uppaal-SMC tool. Initially, Uppaal-SMC focused on networks of priced timed automata. These are timed systems in which real variables may have different rates (even potentially negative) in different modes (these variable are not used as guards). A recent extension allows to handle stochastic hybrid automata; this was first reported in \[36\]. Here, the change of continuous variables is governed by linear differential equations. Uppaal-SMC relies on results from statistics such as sequential hypothesis testing and Monte Carlo simulation. Crucial statistical input parameters are the confidence \(\xi\) which quantifies the error, given by the parameter \(\alpha\) and \(\beta\) (as before), and the probability interval defined by the indifference region \(\delta\). Apart from being able to simulate various variables, we can carry out statistical model checking of temporal logic formulas, in particular, PCTL (probabilistic CTL). This logic allows for specifying (a bound on) the likelihood of reaching a specific set \(T\) of target states within \(n\) steps as denoted by the formula \(P(\Diamond_{\leq n} T)\), or the probability to stay in a given set \(T\) of states for the next time period, denoted by \(P(\Box_{\leq n} T)\).

**Statistical Model Checking Results.** For the microgrid case study we are mainly interested in reachability properties. The first property of interest is the probability of the room temperature to be close to the desired set points within some finite time interval. We vary the value of the time interval in order to investigate the impact of this value on the run time and vary some of the temperature values (set points and desired ones). The model-checking results are listed in Table 2.1. The first column indicates the...
checked property, the second and third column indicate the values of the control inputs (temperate set points). The fourth and fifth column indicate the parameters $\alpha$, $\beta$ and $\delta$ of the SMC algorithm, the sixth column presents the probability interval that results from the SMC, the seventh column indicates the length $t$ of the interval, the eighth column indicates the number of required simulation runs, and the last column presents the run time of UPPAAL-SMC. We present the results of checking three properties. The first property refers to the time until the zone temperature $T_{ZA}$ is at most one degree Celsius different from the desired room temperature $T_{ZASP}$. The second property aims at determining the probability that the chilled water temperature $T_{CW}$ is close to its set point $T_{CWSP}$. As one can see from the results, this probability is low for small $t$ (say, $t \leq 200$), whereas from $t=1,000$, $T_{CW}$ has reached its set point. The last property focuses on the thermal capacity of the water circuit $Q_{cool}$, as realized by a PI controller, reaches a threshold 4,500. We experiment with various combination of control parameters, namely, the room temperature set point of the thermostat ($T_{ZASP}$) and chilled water (cooler) temperature set point ($T_{CWSP}$).

Observe that relaxing the probability interval has a larger effect on the run time than changing the confidence. As we expected lowering of set point temperature makes it longer for the system to attain equilibrium (the simulation figures) and hence the model checker needs to run the simulation for longer duration. This causes an increase in verification time for highly probable properties. On the other hand, the energy requirement

<table>
<thead>
<tr>
<th>PCTL property</th>
<th>$T_{CWSP}$</th>
<th>$T_{ZASP}$</th>
<th>$(\alpha, \beta)$</th>
<th>$\delta$</th>
<th>$P_t$</th>
<th>$t$</th>
<th>runs $(\alpha)$</th>
<th>time $(s)$</th>
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<tr>
<td>$\mathcal{P}(\phi \leq</td>
<td>T_{ZA} - T_{ZASP}</td>
<td>&lt; 1)$</td>
<td>20</td>
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<td>&lt; 1)$</td>
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<td>200</td>
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<td>$\mathcal{P}(\phi \leq Q_{cool} \geq 4500)$</td>
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<td>(0.1, 0.1)</td>
<td>0.1</td>
<td>0.1</td>
<td>200</td>
<td>150</td>
<td>162.83</td>
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<tr>
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<td>0.1</td>
<td>200</td>
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<td>204.51</td>
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<tr>
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<td>0.05</td>
<td>200</td>
<td>600</td>
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<td>0.05</td>
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<td>1000</td>
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<td>1000</td>
<td>738</td>
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<td>0.95</td>
<td>1000</td>
<td>738</td>
<td>10.25</td>
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</tbody>
</table>

Table 2.1: SMC results for temperature control
variable \( Q_{\text{cool}} \), which almost never reached 4500\( J \) now crosses this threshold very quickly because of the high energy demand forced by lowering of set point temperature. Hence, simulations are short and verifications are quite quick.

As a next property, we are interested in the behaviour of the chillers. The PCTL-property of interest is the likelihood that the chiller power demand (of the first chiller, say) exceeds a certain threshold (e.g. 2) within the next 3,000 seconds. The cooling load (CL) is determined by the components outside temperature, zone temperature, thermostat, chilled water temperature, chilled water controller, and the chillers. The results for the first chiller are listed in the upper part of Table 2.2. The lower part of this table presents the results of checking the maximal generated power by the wind turbine to exceed 0.25 kW. Observe that even if the indifference interval is specified to be 0.05, the probability interval given by UPPAAL-SMC actually is larger. We cannot explain this anomaly.

As a next property, we check whether the microgrid in islander mode keeps itself stable or not. We consider the following components: one cooling load (CL), one electrical load (EL), wind (W), five wind turbines (WT) and the local power network (LPN). The LPN crosses dangerous levels, when there is no storage device. Some verification results are listed in Table 2.3 (upper part). Finally, we consider a more complex scenario where it is assumed that the microturbine (MT) was in the warm-up mode, there are two electrical loads, one cooling load with two chillers and two wind turbines. The microgrid goes to the islander mode so as to compensate for the discrepancy the microturbine was speed up to 70000 rpm\(^3\). We check whether the LPN remains stable or not, cf. Table 2.3 (lower part).

Table 2.2: SMC results for cooling load (upper) and maximal wind production (lower)

As a next property, we consider the following components: one cooling load (CL), one electrical load (EL), wind (W), five wind turbines (WT) and the local power network (LPN). The LPN crosses dangerous levels, when there is no storage device. Some verification results are listed in Table 2.3 (upper part). Finally, we consider a more complex scenario where it is assumed that the microturbine (MT) was in the warm-up mode, there are two electrical loads, one cooling load with two chillers and two wind turbines. The microgrid goes to the islander mode so as to compensate for the discrepancy the microturbine was speed up to 70000 rpm\(^3\). We check whether the LPN remains stable or not, cf. Table 2.3 (lower part).

Table 2.3: SMC results for microgrid operating in islander mode

\( ^3 \)Such decisions are taken by network administrators.
2.1.3 ADP based on Q-iteration: A computational approach resting on system simulation and abstraction

We here focus on the control of a small scale building cooling system, consisting of a chiller plant, a small thermal storage unit, and a cooling load, as sketched in Figure 2.6. Similar energy management problems have been addressed e.g. in [62, 61, 14] using model predictive and scheduling techniques.

![Figure 2.6: Configuration of the building cooling system.](image)

The building cooling system has no local power source and fully depends on the main distribution grid for the electric energy supply. The chiller plant converts the electric power provided by the distribution grid into cooling power, which is then conveyed to the cooling load or to the thermal storage through the Chilled Water Circuit (CHWC). The chiller plant includes two chillers characterized by different efficiency curves. The cooling load is associated to the thermal control of a zone, which can be a room, several rooms or a partitioned space in a room. The temperature in the zone is influenced by the outside ambient temperature as well as by internal heat gains mainly due to people occupancy, here described using a probabilistic model.

The “optimal” operation of the building cooling system pursues the two-fold objective of minimizing the electric energy consumption while guaranteeing an adequate comfort level in the zone. To this aim, the chillers must be operated as efficiently as possible, optimizing the cooling power distribution among them, whilst the thermal storage unit can be employed to put some energy aside for later usage. This feature can be exploited to reduce the overall energy consumption, by allowing the chiller plant to operate at efficient constant regimes with the storage unit compensating for request mismatches. The flexibility of the system is further increased by allowing slight and time-limited modulations of the zone temperature set-point, in order to decrease the cooling power request while maintaining an appropriate comfort level. Indeed, significant energy savings can be obtained with a limited variation of the set-point, and, hence, with a limited impact on comfort. In particular, the temperature increment range is designed well within the comfort bounds set by the ISO norm on thermal comfort [51].

Overall, this can be formulated as a constrained stochastic optimal control problem for a Stochastic Hybrid System (SHS) and can be hierarchically decoupled into two sepa-
rate optimization tasks as shown in [33]. Specifically, the power distribution between the chillers results from the solution of a static optimization, which maximizes the overall chiller plant efficiency for a given power request. Then, the cooling power request to the chiller plant and to the thermal storage unit is defined on the basis of a Dynamic Programming (DP) stochastic optimization problem, which also sets the zone temperature set-point variation.

In contrast with [33] where uncertainty is neglected, an approximate solution to the stochastic DP problem is pursued here (see also [19, 20, 18], resting on the abstraction of the underlying SHS to a controlled Markov Chain (MC) with costs associated to transitions computed through appropriately defined simulations of the original hybrid system.

Description of the building cooling system

Plant model

The system does not include any electric generator unit, neither renewable nor traditional, and we assume that the main grid, by way of the Local Power Network (LPN), provides to the chillers the exact amount of electric power required to satisfy the cooling load. The balance equation (2.1) can be rewritten as

\[ P_g = P_{Ch,1} + P_{Ch,2}, \]

where \( P_g \) is the grid power, and \( P_{Ch,i} \) is the electric power request by chiller \( i \).

The energy storage introduced in the Section 2.1.1 can be modified into stratification model. Figure 2.7 shows a two-level stratified model of the thermal storage, where the (cold) lower block is at temperature \( T_c \) and the upper (warm) one at temperature \( T_h \). The cooling energy accumulated in the storage depends on the height \( h_c \) of the cold block, since it is given by \( \rho A_{st} h_c c_p(T_h - T_c) \), where \( \rho \) and \( c_p \) are the water specific density and heat capacity, and \( A_{st} \) is the cross-section area of the storage. Assuming that the total volume of water in the storage is constant, \( h_c \) satisfies

\[ \dot{h}_c = -q_{st}/(\rho A_{st}), \]

where \( q_{st} \) denotes the flow through the storage.

In the charging phase \( (q_{st} < 0) \), the lower block at temperature \( T_c \) is fed by a flow at temperature \( T_{stdw} = T_{ch} \), \( T_{ch} \) being the outlet temperature of the chiller plant, and the outflow from the upper block is at temperature \( T_{stup} = T_h \). In the discharging phase \( (q_{st} > 0) \), the upper block at temperature \( T_h \) is fed by a flow at temperature \( T_{stup} = T_{pipe} \), \( T_{pipe} \) being the temperature at the outlet of the cooling load section of the CHWC, and the outflow from the lower block is at temperature \( T_{stdw} = T_c \).

In this section, the CHWC is split into two parts where one part interacts with chillers and the second one is connected to the zones. Assuming that \( T_{ch} \) is controlled to a constant set-point, and that the heat exchange between the two blocks can be neglected, the lower block stores and releases cold water at \( T_c = T_{ch} \), so that the left-hand-side of the CHWC can be assumed to be at temperature \( T_{ch} \). Similarly, provided that \( T_{pipe} \) is

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controlled to some constant set-point, the right-hand-side of the CHWC can be assumed to be at temperature $T_{pipe}$. This assumption jointly with the condition $q_{pipe} = q_{ch} + q_{st}$ on the flows in the CHWC leads to:

$$C_{pipe} \frac{dT_{pipe}}{dt} = c_p q_{pipe} (T_{ch} - T_{pipe}) + \beta Q_C$$

$$C_{ch} \frac{dT_{ch}}{dt} = c_p q_{ch} (T_{pipe} - T_{ch}) - Q_{Ch},$$

where $C_{pipe}$ and $C_{ch}$ are thermal capacities, $\beta \in \{0, 1\}$ indicates the activation state of the cooling system in the zone, $Q_C$ is the heat power absorbed from the zone (cooling load) when $\beta = 1$, and $Q_{Ch}$ the cooling power provided by the chiller plant.

![Figure 2.7: Scheme of the CHWC.](image)

The cooling load associated with the thermal control of the zone is described through the evolution of its average temperature $T_{ZA}$:

$$C_z \frac{dT_{ZA}}{dt} = -\beta Q_C + Q_i + k_{out}(T_{OA} - T_{ZA}),$$

where $T_{OA}$ (outside ambient temperature) and $Q_i$ (internal heat gain) are disturbances affecting the system, whilst $Q_C = X_C k_{cw} (T_{ZA} - T_{pipe})$ is the heat power released to the pipe, $X_C$ being the position of the thermostat valve. $k_{out}$ and $k_{cw}$ are parameters representing heat transfer coefficients.

The cooling power $Q_{Ch}$ requested to the chiller plant is split between its two chillers as $Q_{Ch,1} = (1 - \alpha)Q_{Ch}$ and $Q_{Ch,2} = \alpha Q_{Ch}$, where $\alpha \in [0, 1]$ is a scheduling parameter to be optimized. The electric power $P_{Ch,i}$ required by chiller $i$ to produce $Q_{Ch,i} \in [0, Q_{Ch,i}]$ is given by nonlinear static model (2.5).
If chiller \( i \) is off, then \( P_{Ch,i} = 0 \). The efficiency of the chiller plant can be characterized through the coefficient of performance

\[
COP = \frac{Q_c}{P_{Ch,1} + P_{Ch,2}},
\]

which is a static function of \( \alpha, Q_{Ch}, T_{pipe}, \) and \( T_{OA} \).

Temperature controllers

Temperatures \( T_{pipe} \) and \( T_{ZA} \) are kept at the set-points \( T_{\star_{pipe}} \) and \( T_{\star_{ZA}} \) by means of two PI controllers acting on \( q_{pipe} \) and \( X_C \), respectively. In particular, maintaining \( T_{pipe} \) at some constant value \( T_{\star_{pipe}} \) facilitates the stratification in two blocks of the thermal storage. The zone temperature set-point \( T_{\star_{ZA}} \) is given by \( T_{\star_{ZA}} = \bar{T}_{ZA} + \Delta_{\star_{ZA}} \), where \( \Delta_{\star_{ZA}} \in [0, \Delta_{\max}] \) is the variation with respect to the reference set-point value \( \bar{T}_{ZA} \) that is allowed to save energy.

Temperature \( T_{ch} \) is maintained at some constant set-point \( T_{\star_{ch}} \) through the following switching control scheme.

If the storage is not available, then \( q_{st} = 0 \) (and, hence, \( q_{ch} = q_{pipe} \)) and \( T_{ch} \) is kept at \( T_{\star_{ch}} \) by a PI controller with disturbance compensation acting on \( Q_{c} \):

\[
Q_c = k_{p}^{ch} (T_{ch} - T_{\star_{ch}}) + k_{i}^{ch} \int (T_{ch} - T_{\star_{ch}}) dt + q_{ch} c_{p} (T_{pipe} - T_{ch});
\]

otherwise, the chiller plant is assigned some (constant) cooling power request \( Q_{\star_{c}} \in [0, Q_{\max}^c] \) and the storage eventually compensates for the residual cooling power needed to keep \( T_{ch} \) equal to \( T_{\star_{ch}} \). In this latter case, the flow through the chiller plant would be given by

\[
q_{ch} = q_{\star_{ch}} = k_{p}^{ch} (T_{ch} - T_{\star_{ch}}) + \frac{Q_{\star_{c}}}{c_{p} (T_{pipe} - T_{ch})},
\]

thus requiring a flow \( q_{st} = q_{\star_{st}} = q_{pipe} - q_{ch} \) through the storage. If we denote the height of the storage by \( h_{st} \), then its availability represented by the binary variable \( a_{st} \in \{0, 1\} \) can be expressed as \( a_{st} = (0 < h_{c} < h_{st}) \lor (h_{c} = h_{st} \land q_{\star_{st}} \geq 0) \lor (h_{c} = 0 \land q_{\star_{st}} \leq 0) \). The adopted switching logic guarantees that the constraint \( 0 \leq h_{c} \leq h_{st} \) is satisfied.

Disturbances

The building cooling system is subject to two disturbances, i.e., the internal heat gain and the outside ambient temperature.

In this work, the internal heat gain \( Q_i \) is modeled as the product of the internal heat generated by a single person with the number \( n_P \) of occupants of the zone:

\[
Q_i = \left[a_1 T_{ZA}^2 + a_2 T_{ZA} + a_3\right] n_P.
\]

Indeed, occupants constitute a significant source of heating in densely occupied buildings, such as offices and shops, and, due to the improved building thermal insulation, they are becoming an even more important factor. \( n_P \) is modeled through a birth-death process with time varying birth (arrivals) and death (departure) rates. This can be
viewed as a generalization of the model in [81], where a Markov chain is employed to model a single occupant.

It is assumed that the building is inhabited only during the day and people start entering the building at a specified time $t_{in}$. Accordingly, we define $n_P$ as follows:

$$n_P(t) = \max (n_P^{in}[t_{in}, t] - n_P^{out}[t_{in}, t], 0),$$

where $n_P^{in}[t_{in}, t]$ and $n_P^{out}[t_{in}, t]$ are independent Poisson processes representing respectively the number of arrivals and departures within $[t_{in}, t]$. The time-varying rates $\lambda_{in}()$ and $\lambda_{out}()$ of $n_P^{in}[t_{in}, t]$ and $n_P^{out}[t_{in}, t]$ are defined based on a nominal occupancy profile $\bar{n}_P$ which is nonzero in a given time interval $[t_{in}, t_{out}]$. Specifically, observing that

$$E[n_P^{in}[t_{in}, t] - n_P^{out}[t_{in}, t]] = \int_{t_{in}}^{t} \lambda_{in}(\eta) d\eta - \int_{t_{in}}^{t} \lambda_{out}(\eta) d\eta,$$

we define the rates within $[t_{in}, t_{out}]$ based on the time derivative $\dot{\bar{n}}_P$ of the nominal occupancy profile as follows:

$$\lambda_{in} = \begin{cases} \dot{\bar{n}}_P, & \dot{\bar{n}}_P > 0 \\ 0, & \dot{\bar{n}}_P \leq 0 \end{cases} \quad \lambda_{out} = \begin{cases} -\dot{\bar{n}}_P, & \dot{\bar{n}}_P < 0 \\ 0, & \dot{\bar{n}}_P \geq 0 \end{cases}.$$

Further, after $t_{out}$, the $\lambda_{out}$ rate is set to a sufficiently high value so as to guarantee with probability 0.99 that the building is empty within one hour.

Figure 2.8 plots some realizations of $n_P$ given some nominal profile $\bar{n}_P$.

![Figure 2.8: Some realizations of the occupancy profiles. The nominal profile with $[t_{in}, t_{out}] = [7, 20]$ is represented as a dashed line. Different colors are used to distinguish different realizations.](image)

The outside temperature $T_{OA}$ is assumed to be given by some accurate forecast and treated as a deterministic signal. Indeed if the insulation level of the building is high, fluctuations around the forecast value have a limited impact and the effect of the internal heat gain is dominant.
Remark 1 Note that the described stochastic system is hybrid since it comprises both continuous state variables \((T_{ZA}, T_{ch}, T_{pipe}, X_C, q_{pipe}, Q_c, h_c)\) and discrete state variables \((n_P, a_{st})\). The control variables available to the energy management system are given by the chiller plant scheduling parameter \(\alpha \in [0, 1]\) and the zone temperature and cooling power modulation variables \(\Delta_{ZA}^* \in [0, \Delta_{max}]\) and \(Q_c^* \in [0, Q_{max}]\). An additional state variable

\[
d(t) = \int_{t_0}^{t} \Delta_{ZA}^*(t) dt
\]  

(2.14)

is introduced to account for the discomfort caused by the modulation of the zone temperature set-point within \([t_0, t]\).

Optimal energy management problem

Given some reference time horizon \([t_0, t_f]\), our goal is to determine a policy

\[\pi : \mathcal{S} \times [t_0, t_f] \rightarrow [0, 1] \times [0, \Delta_{max}] \times [0, Q_{max}],\]

that maps the state \(s \in \mathcal{S}\) of the system and the current time instant \(t \in [t_0, t_f]\) into a value for \(\alpha, \Delta_{ZA}^*, Q_c^*\) so as to minimize the average electric energy consumption, while limiting the discomfort caused by the zone temperature set-point modulation. This translates into the constrained stochastic optimization problem:

\[
\min_{\pi} E_{\pi_0}^\pi \left[ \int_{t_0}^{t_f} P_g(t) dt \right] \text{ subject to: } d(t_f) \leq d_{max},
\]

where \(d\) is the discomfort state variable defined in (2.14) and \(d_{max}\) is the maximum allowed discomfort level that can be reached at time \(t_f\).

Following [33], the problem can be decomposed without loss of optimality into two phases:

- design \(\pi_\alpha : \mathcal{S} \times [t_0, t_f] \rightarrow [0, 1]\) for scheduling the chillers, and
- design \(\pi_{\Delta_{ZA}^* Q_c^*} : \mathcal{S} \times [t_0, t_f] \rightarrow [0, \Delta_{max}] \times [0, Q_{max}]\) for the set-point and the chiller cooling power request modulation.

The first phase reduces to a static optimization problem where \(\alpha\) is chosen so as to minimize \(P_g\) for each given triple \((Q_c, T_{OA}, T_{pipe})\) (see [33] for more details). The extension to the case of a chiller plant with possibly more than 2 chillers is treated in [20], where a solution to the static scheduling problem that is scalable in the number of chillers is introduced.

Based on the resulting optimal \(P_g^*\), the modulation policy \(\pi_{\Delta_{ZA}^* Q_c^*}\) is computed by solving

\[
\min_{\pi_{\Delta_{ZA}^* Q_c^*}} E_{\pi_0}^{\pi_{\Delta_{ZA}^* Q_c^*}} \left[ \int_{t_0}^{t_f} P_g^*(t) dt \right] \text{ subject to: } d(t_f) \leq d_{max}.
\]

(2.15)
If $\Delta^*_{ZA}$ and $Q^*_c$ can take only a finite set of values, say $\mathcal{U}_c$ and $\mathcal{U}_e$, and are not modulated continuously but every $\tau = \frac{t_k - t_0}{N}$ minutes, this can be rephrased as a finite-horizon control problem for a discrete time SHS with a discrete control input set. The executions of this SHS are obtained by sampling the executions of the original continuous time SHS with the control inputs $\Delta^*_{ZA}$ and $Q^*_c$ held constant over each time frame $[\tau_k, \tau_{k+1}]$ with $\tau_k := t_0 + k\tau$. The problem can be tackled through DP techniques, with the state constraint reformulated as a constraint on the admissible values for the control input $\Delta^*_{ZA}$ based on the residual discomfort.

The numerical solution to the resulting DP equations is hampered by the presence of continuous state components and expectation with respect to the birth-death process $n_P$. The idea developed here is to find an Approximate DP (ADP) solution by abstracting the underlying SHS to an inhomogeneous (finite state) controlled MC, with costs associated to transitions computed through appropriately defined simulations of the original hybrid system. The quality of the approximate abstraction can be assessed through randomized techniques, as shown in [20] for a configuration of the building cooling system without thermal storage.

The idea of adopting a finite approximate abstraction of the system to address the DP solution is inspired by the hierarchical approach in [22]. A key difference with respect to [42] is that here we deal with stochastic hybrid systems for which only few results on the design of approximate abstractions with provable approximation guarantees are available, [52] [53] [4] [41] [88].

**ADP solution based on MC abstraction**

The SHS is abstracted to a controlled MC with state set $\mathcal{X}$, control set $\mathcal{U} = \mathcal{U}_c \times \mathcal{U}_e$, and controlled transition probability function $p_k : \mathcal{X} \times \mathcal{U} \times \mathcal{X} \to [0,1], k = 0, 1, \ldots, N - 1$. State $x = (T_{ZA}, d, h_c, n_P) \in \mathcal{X}$ accounts for variables $T_{ZA}$, $d$, $h_c$, and $n_P$ of the original SHS at the sample times $\tau_k$, $k = 0, 1, \ldots, N$. Its components take values in suitably defined finite sets. In particular, when $\beta = 1$, $\tilde{T}_{ZA} \in \{T_{ZA} + \Delta_{ZA}^*: \Delta_{ZA}^* \in \mathcal{U}_c\}$, $d \in \{\tau_k \Delta_{ZA}^*: k = 0, 1, \ldots, N$, $\Delta_{ZA} \in \mathcal{U}_e\} \cap [0,d_{\max}]$, $h_c \in \{i \cdot \delta h_{st}: i = 0, 1, \ldots, \hat{h}_c\}$, and $n_P \in [n_{P_{\min}},n_{P_{\max}}]$, where $n_{P_{\min}}$ and $n_{P_{\max}}$ are chosen so that 99% of the occupancy profiles keep within $[n_{P_{\min}},n_{P_{\max}}]$.

The probability $p_k(x,u,x')$ that the MC evolves from $x = (\tilde{T}_{ZA}, \tilde{d}, h_c, \hat{n}_P)$ at time $k$ to $x' = (\tilde{T}_{ZA}, \tilde{d}', h'_c, \hat{n}'_P)$ at time $k+1$ depends on the control action $u = (\Delta_{ZA}^*, Q^*_c) \in \mathcal{U}$ that is applied at $k$, and is zero if $x'$ is not admissible as next state. In particular, $\tilde{T}_{ZA}$ must satisfy $\tilde{T}_{ZA} = \tilde{T}_{ZA} + \Delta_{ZA}^*$ (since temperature $T_{ZA}$ is controlled to $T_{ZA}^* = T_{ZA} + \Delta_{ZA}^*$); $\tilde{d}' = \tilde{d} + \Delta_{ZA}^* \tau$ (based on the definition of discomfort); $h_c'(\tau_{k+1})$ obtained when the SHS evolves within $[\tau_k, \tau_{k+1}]$ from $T_{ZA}(\tau_k) = \tilde{T}_{ZA}$, $d(\tau_k) = \tilde{d}$, $h_c(\tau_k) = h_c$, and $n_P(\tau_k) = \hat{n}_P$ (with the other state variables set at consistent equilibrium values), subject to the linear occupancy profile obtained by joining $\hat{n}_P$ to $\hat{n}'_P$, the outside ambient temperature $T_{OA}(t)$, $t \in [\tau_k, \tau_{k+1})$, and the control inputs $\Delta_{ZA}^*$ and $Q^*_c$. If $\tilde{T}_{ZA}$, $\tilde{d}'$, and $h'_c$ satisfy these conditions, then, $p_k(x,u,x')$ equals the probability of having $n_{P'} - n_{P}$ arrivals/departures within $[\tau_k, \tau_{k+1}]$, otherwise is zero. To have $p_k(x,u,x')$ well
defined as a probability, i.e., summing up to 1 when \( \hat{n}_P \) ranges within \([n_P^{\text{min}}, n_P^{\text{max}}]\), we assign to the extreme values for \( \hat{n}_P \) the probability associated to all arrivals/departures \( \Delta_P \) within \([n_{\text{min}} P, n_{\text{max}} P]\), we assign to the extreme values for \( \hat{n}_P \) the probability associated to all arrivals/departures \( \Delta_P \) within \([n_{\text{min}} P, n_{\text{max}} P]\).

Given the controlled MC abstraction of the original SHS, we need to associate a cost \( \hat{c}_k(x, u, x') \) to each admissible transition from \( x \) to \( x' \) when the control input \( u \) is applied at time \( k \). This cost represents the electric energy consumption for that transition and can be determined by simulating the original SHS within \([\tau_k, \tau_{k+1}]\) as described above when defining the admissible values for \( \hat{h}_c' \).

Problem (2.15) then reduces to determining policy \( \nu : X \times [0, N - 1] \to U \) by solving

\[
\min_{\nu} E_{x_0}^{x_N} \left[ \sum_{k=0}^{N-1} \hat{c}_k(x_k, u_k, x_{k+1}) \right] \text{ subject to: } \hat{d}_N \leq d_{\text{max}}.
\]

The optimal policy can be computed as follows

\[
\nu(x, k) \in \arg \min_{(\Delta_{ZA}', Q_c') \in \mathcal{U}_z(\hat{d}) \times \mathcal{U}_A} V_k(x, (\Delta_{ZA}', Q_c')),
\]

where \( V_k : X \times U \to \mathbb{R}_+ \), \( k = 0, 1, \ldots, N - 1 \), are the so-called Q-functions and \( \mathcal{U}_z(\hat{d}) \) is the admissible set of values for \( \Delta_{ZA}' \) when the discomfort value is \( \hat{d} \). The Q-functions can be computed according to the DP equations:

\[
V_k(x, u) = \sum_{x' \in X} p_k(x, u, x') \hat{c}_k(x, u, x') + \min_{u' \in \mathcal{U}_c(\hat{d})} V_{k+1}(x', u'),
\]

for \( k = 0, 1, \ldots N - 2 \), initialized at \( k = N - 1 \) with

\[
V_{N-1}(x, u) = \sum_{x' \in X} p_{N-1}(x, u, x') \hat{c}_{N-1}(x, u, x').
\]

**Numerical example**

The proposed ADP-based approach is applied to the energy management of a building cooling system along a one-day time horizon \([t_0, t_f] = [0, 24] \text{ hours}\). The zone is occupied from 7 to 21 hours, according to the stochastic occupancy profile described before, and is cooled from 6 to 22 hours, when \( \beta = 1 \). The outside ambient temperature \( T_{OA} \) is given by the forecast in Figure 2.9.

Zone temperature set-point and cooling power request to the chiller plant can be changed every \( \tau = 30 \text{ minutes} \). We assume that \( \Delta_{ZA}' \in \mathcal{U}_z = \{0, \Delta_{\text{max}}/2, \Delta_{\text{max}}\} \), with \( \Delta_{\text{max}} = 1^\circ\text{C} \) and \( Q_c' \in \mathcal{U}_A = \{k \delta Q_c : k = 0, 1, \ldots, 12\} \), where \( \delta Q_c = 2 \text{ kW} \). The maximum discomfort level is \( d_{\text{max}} = 6^\circ\text{C} \), corresponding to an increase of \( 1^\circ\text{C} \) for 6 hours.

A list of the system parameter values is given in Table 2.4.

**Chiller plant optimization**

Figure 2.10 shows the COP of the chiller plant as a function of the requested cooling power \( Q_C \) and of the outside ambient temperature \( T_{OA} \) for \( T_{\text{pipe}} = T'_{\text{pipe}} = 15^\circ\text{C} \), when...
<table>
<thead>
<tr>
<th>Zone</th>
<th>$C_z$</th>
<th>$k_{out}$</th>
<th>$T_{ZA}$</th>
<th>$k_p$</th>
<th>$k_{i,z}$</th>
<th>$T^*_{ZA}$</th>
<th>$T_{ZA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$6092 \cdot 10^4 , J , ^\circ C^{-1}$</td>
<td>$462.5 , W , ^\circ C^{-1}$</td>
<td>$9.25 , ^\circ C^{-1}$</td>
<td>$0.0025 , ^\circ C^{-1} \cdot s^{-1}$</td>
<td>$20 , ^\circ C$</td>
<td>$3.1 \cdot 10^3 , W , ^\circ C^{-1}$</td>
<td>$2.22 \cdot 10^4 , J , ^\circ C^{-1}$</td>
</tr>
<tr>
<td>Thermal</td>
<td>$C_{st}$</td>
<td>$h_{st}$</td>
<td>$\delta h_{st}$</td>
<td>$19.2 , J , ^\circ C^{-1}$</td>
<td>$3 , m$</td>
<td>$0.03 , m$</td>
<td>$19.2 , J , ^\circ C^{-1}$</td>
</tr>
<tr>
<td>CHWC</td>
<td>$C_{ch}$</td>
<td>$k_{ch}$</td>
<td>$k_{pipe}$</td>
<td>$k_i$</td>
<td>$k_{pipe}$</td>
<td>$T^*_{pipe}$</td>
<td>$T_{pipe}$</td>
</tr>
<tr>
<td></td>
<td>$1.31 \cdot 10^6 , J , ^\circ C^{-1}$</td>
<td>$1.31 \cdot 10^6 , J , ^\circ C^{-1}$</td>
<td>$3.29 \cdot 10^3 , W , ^\circ C^{-1}$</td>
<td>$-14.4 , kg \cdot s^{-1} \cdot ^\circ C^{-1}$</td>
<td>$-14.4 , kg \cdot s^{-1} \cdot ^\circ C^{-1}$</td>
<td>$15 , ^\circ C$</td>
<td>$15 , ^\circ C$</td>
</tr>
<tr>
<td>Chiller 1</td>
<td>$a_{1,1}$</td>
<td>$a_{1,2}$</td>
<td>$a_{1,3}$</td>
<td>$a_{1,4}$</td>
<td>$0.0056 , W , ^\circ C^{-1}$</td>
<td>$10.11 , W$</td>
<td>$7 , ^\circ C \cdot W^{-1}$</td>
</tr>
<tr>
<td>Chiller 2</td>
<td>$a_{2,1}$</td>
<td>$a_{2,2}$</td>
<td>$a_{2,3}$</td>
<td>$a_{2,4}$</td>
<td>$0.0109 , W , ^\circ C^{-1}$</td>
<td>$20.22 , W$</td>
<td>$3.807 , ^\circ C \cdot W^{-1}$</td>
</tr>
<tr>
<td>Internal heat</td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$a_3$</td>
<td>$-0.2199 , W , ^\circ C^{-2}$</td>
<td>$5.0597 , W , ^\circ C^{-1}$</td>
<td>$84.9168 , W$</td>
<td>$84.9168 , W$</td>
</tr>
</tbody>
</table>

Table 2.4: List of system parameters
the optimal scheduling parameter $\alpha$ is adopted. Both chillers provide a maximum cooling power supply $Q_{\text{max}}$ of 30 kW, but with quite different efficiency curves, as shown in the top plot of Figure 2.10 (chiller 1 performs better for low cooling power values, whereas chiller 2 at higher power values), resulting in the complex scheduling function.

![Figure 2.9: Outside ambient temperature](image)

![Figure 2.10: Plot on the top: COP of the two chillers. Plot on the bottom: Optimal COP of the chiller plant.](image)

**ADP solution**

The policy obtained with the ADP-based approach results in the chiller power request profile shown in Figure 2.11 when applied to the nominal occupancy profile. For reference
purposes, the control in the absence of thermal storage is also shown. Notice that, in the
latter case, a huge amount of cooling power is required at the onset of the cooling period
to bring the zone temperature at its set-point. Afterwards, the power request to the
chillers tracks the cooling power demand, thus bringing occasionally the chillers to work
in non-efficient operating regions. On the other hand, when the thermal storage unit is
employed, the power request to the chillers can be kept piecewise constant most of the
time, at levels that allow the chiller plant to work at operating points where its COP is
higher. The charging of the thermal storage is scheduled (by the computed policy) early
in the morning and then supplies cooling energy together with the chiller plant. More
specifically, it compensates for power mismatches, given that the chiller plant is driven
on purpose at a constant power level for better efficiency, and is effective in this task for
most of the day. When the storage is exhausted the power request to the chiller plant
cannot be kept constant anymore and follows the actual request.

![Figure 2.11: Cooling power requested to the chiller plant by the optimal policy in the
nominal occupancy case.](image)

The corresponding behavior of the zone temperature is reported in the top plot of
Figure 2.12 together with the zone temperature set-point as modulated by the computed
policy. Given that the zone is cooled only from 6 to 22, its temperature is uncontrolled
outside that time interval. Notice that most of the temperature set-point modulation
occurs between 10:30 and 15:30, where the occupancy profile displays its peaks. A further
set-point modulation appears to be necessary when the storage is exhausted.

It is also important to assess the cold water storage during the system operation (see
Figure 2.12). Charging takes place essentially in the early hours of the day, when the
building is empty, but, interestingly enough, after a brief discharge transient coincident
with the activation of the cooling phase, the storage unit is further charged to its maxi-
imum level to be used afterwards in the peak hours. Indeed, it progressively discharges
during the remaining office hours, complementing the cooling power provided by the
chiller plant.
To assess the control system performance in terms of energy saving, Monte Carlo simulations were carried out considering 10000 occupancy profiles, extracted independently according to the probabilistic model described before. A reference policy was computed for comparison purposes, where only the chiller scheduling is optimized, while the temperature set-point is set constant to 20°C and the thermal storage is not available (so that the chillers must always supply the exact amount of requested cooling power). On average, the proposed method consumes 73.38 kWh, compared to the 78.02 kWh energy consumption of the reference policy, for an average energy saving of 5.94%. This illustrates the potential benefits of temperature set-point modulation and thermal storage. Another interesting comparison is with a policy exploiting both these features, but computed neglecting uncertainty and considering only the nominal occupancy profile, as in [33]. Such a policy has an average cost of 76.85 kWh, clearly demonstrating the superiority of the presented stochastic approach, which achieves a 4.51% power reduction.
2.1.4 ADP based on Policy Search

Introduction

MG energy management problem has been widely investigated, e.g. in [111], [105] and others. We focused on the application of dynamic programming approach to MG optimization where approximate dynamic programming stands for promising method to overcome curse of dimensionality problem [87]. Opposite to previous section, this section focuses on the policy search approach which directly searches for an optimal policy based on cost function approximation.

Microgrid Model

We consider small scale MG which is depicted on the Figure 2.13. Distribution Grid is the major electricity source which is supported by local generator – Wind Turbine. Both sources are connected to the Local Power Network (LPN) which interconnects electricity supply and demand. Several Chillers (Chiller Plant) stand for electricity loads and produce thermal energy, which supplies the Chilled Water Circuit (CHWC). Thermal energy from CHCW is utilized in the Cooling load and can be stored in the Thermal Storage which directly influences cooling demand and therefore indirectly affects electricity demand. All elements are deterministic except Wind Turbine and Cooling Load.

\[ P_{WT}(t) + P_G(t) = \sum_{i=1}^{3} P_{Ch,i}(t), \quad (2.16) \]

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where $P_W$ is power generated by wind turbine, $P_G$ is power loaded from the main distribution grid, $P_{Ch,i}$ is power consumed by $i$-th chiller.

**Wind Turbine**

Wind turbine is modeled as nonlinear static model described by the equation (2.2). In this section, we considered properties of the wind turbine Aeolos–H, which has maximal power output 5 kW. The figure Fig. 2.14 captures the power curve of given wind turbine.

![Power curve of Aeolos–H 5kW](image)

Figure 2.14: Power curve of the Aeolos – H 5kW

More details about wind turbine can be found in [7].

**Chiller**

Model of the chiller is given by the equation (2.4) where the parameters $a(\cdot),\{1,2,3,4\}$ are estimated from measured data or given by a table. In this section, we consider three chillers with various COP curves. These chiller COP curves were generated artificially for experimental purposes. Performance of the chillers can vary from 0 kW to 30 kW. Figure 2.15 shows the dependency of the COP on the outside temperature $T_{OA}$ and chiller duty $Q_{Ch}$.

![COP dependency on outside temperature and chiller duty](image)

Chilled Water Circuit

Chilled water circuit (CHWC) interconnects thermal energy supplies with thermal loads and its model is defined by the equation (2.9). Temperature of the chilled water temperature is considered as $T_{CW} = 10^\circ C$.

**Thermal Energy Storage**

Thermal energy storage model is defined by the equation (2.10) and considered thermal storage capacity is $Q_{ES,\text{MAX}} = 20$ kWh.

**Cooling Load**

Black box approach to modelling of the cooling load is considered here. The load behavior is given by the stochastic differential equation similar to (2.6).

**Outside Temperature**

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Outside temperature evolves according to the stochastic differential equation (2.13).

**Problem Formulation**

Formulation of the Microgrid optimization problem is introduced in this section. At first, energy balance equations are defined. Afterwards two optimization tasks are formulated – Economic Dispatch and Unit Commitment.

**Balance equations**

During the optimization, balance equations for electrical and thermal energy part have to be satisfied. Electrical balance can be defined as follows:

\[
P_W(t) + P_G(t) = \sum_{i=1}^{3} P_{Ch,i}(Q_{Ch,i}(t), T_{OA}(t)),
\]

where \(P_W\) is power generated by wind turbine, \(P_G\) is power loaded from the grid, \(P_{Ch,i}\) is power consumed by \(i\)-th chiller which depends on chiller’s duty \(Q_{Ch,i}\) and outside temperature \(T_{OA}\). Chiller duty as well as outside temperature influence the efficiency of particular chiller. Thermal energy balance equation is formulated as

\[
Q_{Ch}(t) = \sum_{i=1}^{N} Q_{Ch,i}(t) = Q_C(t) - Q_E(t),
\]

where \(Q_L\) is cooling load, \(Q_S\) is energy loaded (stored) from (into) thermal storage, \(Q_{Ch}\) is overall chiller plant duty which is the sum of duties of particular chillers \(Q_{Ch,i}\).
Economic Dispatch

Economic dispatch decides about optimal power output for all running chillers. The solution of economic dispatch determines operating points of running devices that minimize overall operation costs while energy demands are satisfied.

For considered MG, the optimization problem can be formulated as

\[
\min P_{Ch}(t) = \min_{k_i, i = 1, \ldots, N} \sum_{i=1}^{N} P_{Ch,i}(k_i \cdot Q_{Ch}(t), T_{OA}(t)),
\]

subject to

- Energy balance constraints given by Equations (2.17) and (2.18) where duty of \(i\)-th chiller is \(Q_{Ch,i} = k_i \cdot Q_{Ch}\) and \(\sum_{i=1}^{N} k_i = 1\).
- Capacity constraints define capacity limits for operating point of each chiller \(P_{min,i} \leq P_{Ch,i} \leq P_{max,i}\) where \(P_{min,i}\) and \(P_{max,i}\) represent minimal and maximal values for given operating point \(P_{Ch,i}\).
- Storage constraints defines the capacity of the thermal storage where \(Q_{S,min} \leq Q_{S} \leq Q_{E,max}\). \(Q_{E,min}\) is minimal and \(Q_{E,max}\) represent maximal storage energy level.

Unit Commitment

The unit commitment determines operation schedule of the devices and defines their start up and shut down times. Minimization of the objective function incorporating formulation of both economic dispatch and unit commitment along the whole solution horizon \(T\) can be written as follows:

\[
\min_{u(t)} \sum_{t=1}^{T} \left[ c(t) \cdot (P_{Ch}(t) - P_{W}(t)) + \sum_{i=1}^{N} X_i(t) \cdot \left( (1 - X_i(t-1)) \cdot C_{i}^{start} + C_{i}^{run} \right) \right],
\]

subject to Energy balance, Capacity and Storage constraints defined in previous section. Variable \(c\) is electricity price, \(P_{Ch}\) is power loaded by chiller plant\(^4\), \(P_{W}\) is power generated by wind turbine, \(X\) is binary decision variable, \(C_{i}^{start}\) represents costs related to start of \(i\)-th chiller and \(C_{i}^{run}\) are costs related to the operation of \(i\)-th chiller.

Approximate Dynamic Programming based on Policy Search

Approximate dynamic programming (ADP) is a technique known from several technical fields (e.g. \([87], [15]\) or \([97]\)). ADP uses Bellman equation which can be written in

\(^4\)Value of \(P_{Ch}\) is optimized in the economic dispatch task.

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expectation form as follows:

$$V_t(x(t)) = \min_{u(t)} \mathbb{E}_\omega \left\{ C(x(t), u(t), \omega(t)) \right\}$$

$$+ \gamma V_{t+1}(x(t+1)|x(t), \omega(t+1)),$$

(2.21)

where $x$ is the state vector, $u$ is action vector, $\omega$ is the vector of random variables, $\gamma$ is the discount factor, $C$ is the cost function and $V$ stands for cost-to-go function.

MG can be modeled via stochastic hybrid system where continuous variables evolve according to ODE and discrete dynamics can be captured by discrete automata [96]. For considered MG, we define hybrid state space $x(t)$ as follows:

$$x(t) = \left( P_W(t), Q_C(t), Q_S(t), T_{OA}(t), T_{CW}(t) \right),$$

(2.22)

where $P_W$ is wind turbine power, $Q_C$ is thermal (cooling) load, $Q_S$ is the energy stored in thermal storage, $T_{OA}$ is outside temperature and $X_{i=1,\ldots,N}$ is commitment of $i$-th chiller.

Decision vector $u(t)$ can be defined as follows

$$u(t) = \left( Q_E(t+1), X_1(t+1), \ldots , X_N(t+1) \right),$$

(2.23)

and determines the energy flow from/to thermal storage $Q_E$ and chiller commitments $X_i$ for next time instance $t+1$. Vector of random variables $\omega(t) = (P_W(t), T_{OA}(t), Q_C(t))$ is included in the state space and its evolution is driven by stochastic differential equations.

We adopted the approach based on policy search method which uses optimization techniques to directly search for an optimal policy, which minimizes costs from every initial state. The optimization criterion should be therefore a combination (e.g. average) of the costs (returns) from an optimal policy [25]. The computation of the actual value of the cost function $C(x(t), u(t), \omega(t))$ in the Bellman equation (2.21) will be replaced by its estimate $\hat{C}(f)$ where $f$ is vector of the features. From the state space vector (2.22), vector of the features $f$ can be derived as

$$f(t) = \left( P_W(t), Q_C(t), T_{OA}(t), T_{CW}(t) \right)^T.$$

(2.24)

Algorithm 2.1.1 describes modified basic ADP algorithm. At first, the approximation structure is initialized and the initial state of the system is selected (Step 0.). From the predefined set of scenarios [6] one scenario is selected. For the selected scenario $\omega^n$, vector of actions is optimized (Step 2). Actual value of the cost function $C$ is not computed, but the approximate cost function $\hat{C}$ is used instead. Afterwards the system is simulated (Step 3) and the approximation architecture $\hat{C}$ is updated based on the simulation results which contain actual values of the cost function $C(x(t), u(t), \omega(t))$. Until all realizations are optimized (Step 4), algorithm jumps to Step 1. Finally, optimized vector of actions $u^*$ for selected forecasts $\omega^j$ is computed based on updated approximation structure $\hat{C}$.

---

Hybrid state space consists of continuous and discrete variables.

Scenario is considered as one realization of random variables vector $\omega$.
Algorithm 2.1.1 ADP algorithm using a cost function approximation.

**Step 0.** Initialization of the approximation structure $\hat{C}(f)$
- **Step 0a.** Initialize $\hat{C}(f)$ for selected values of the feature vector $f$
- **Step 0b.** Choose an initial state $x_0$
- **Step 0c.** Set $n = 1$

**Step 1.** Choose sample realization of random variables $\omega^n$.

**Step 2.** Solve optimization problem
$$V_t(x(t)) = \min_{u^n(t)} \{ \hat{C}(f) + V_{t+1}(x(t+1)|x(t),\omega^n(t+1)) \}$$

**Step 3.** Perform simulation
- **Step 3a.** Simulate system with the optimized actions $u^n(t)$
- **Step 3b.** Store simulation results and update the approximation architecture $\hat{C}(f)$.

**Step 4.** Let $n = n + 1$. If $n < N$, goto step 1.

**Step 5.** Compute optimized actions $u^*$ (steps 2, 3, 4) based on selected forecast of random variables $\omega^f$.

Two approximation architectures will be used for approximation of the cost function – Look-up table and Receptive Field Weighted Regression.

**Look-up Table Representation**
Look-up table stands for basic approximation structure. Features vector $f$ determines coordinates in the table and the values in particular cells can be aggregated based on selected aggregation function. Following equation expresses aggregation function based on moving average:
$$\hat{C}(f) = (1 - \alpha) \cdot \hat{C}(f) + \alpha \cdot C(x(t), u(t), \omega(t)) , \quad (2.25)$$
where $\alpha$ is forgetting factor and $C(x(t), u(t), \omega(t))$ is actual value of the cost function. The actual value is computed during solution of the economic dispatch task. Another aggregation function to be analyzed is the maximum function which is defined as
$$\hat{C}(f) = \max \left( \hat{C}(f), C(x(t), u(t), \omega(t)) \right) . \quad (2.26)$$

**Receptive Field Weighted Regression**
The idea of receptive field weighted regression (RFWR) is described in [90]. Feature space is approximated by set of local polynomial models which parameters are identified continuously. Parameters of particular model are updated based on distance between current state and the center of the area where the model is sensitive. Differential vector $\tilde{f}_i$ expresses the difference between current value of feature vector and center of the sensitive area. This vector can be computed as
$$\tilde{f}_i = f - c_i , \quad (2.27)$$

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where $f_i$ is the feature vector and $c_i$ stands for center of the sensitive area for $i$-th model.

Basis function $\phi_i(\tilde{f}_i)$ representing $i$-th model is defined as

$$\phi_i(\tilde{f}_i) = a_{0,i} + a_{1,i} \cdot \tilde{f}_{1,i} + \ldots + a_{M,i} \cdot \tilde{f}_{M,i},$$

where $a_{(j),i}$ are regression parameters of $i$-th model. The parameters are updated continuously based on a recursive identification method (e.g. [110]), where features vector $f(t)$ represents the regressor vector and actual value of the cost function $C(x(t), u(t), \omega(t))$ stands for the measurement. Weighting coefficient $\theta_i$ related to $i$-th receptive field can be computed as $\theta_i(\tilde{f}) = \exp(-\frac{1}{2} \tilde{f}^T D_i \tilde{f})$, where $D_i$ represents a bandwidth of $i$-th receptive field. Overall cost function $\hat{C}(f)$ is defined as weighted sum of perceptive fields values

$$\hat{C}(f) = \frac{\sum_{i=1}^{K} \theta_i(\tilde{f}_i) \cdot \phi_i(\tilde{f}_i)}{\sum_{i=1}^{K} \theta_i(\tilde{f}_i)},$$

where $k$ is number of receptive fields, $\theta$ is weight coefficient and $\phi$ stands for basis function.

**Experimental Results**

The techniques described above were tested on simulation model described in the Section 2.1.4. A number of different profiles of cooling demand, outside air temperature and wind speed were simulated on the model whose devices were operated based on commitments computed by particular optimization techniques – full dynamic programming approach, a myopic strategy without storage utilization, ADP with three variants of the approximation architecture.

Myopic strategy computes the optimal unit commitment and economic dispatch at each time $t$ and therefore cannot utilize the thermal storage. Full dynamic programming computes cost function $C(x, u, \omega)$ always while ADP works mostly with cost function approximation $\hat{C}(f)$ and computes cost function $C(x, u, \omega)$ according to the Algorithm 1. For simplicity, all strategies do not consider costs related to start and operation of chillers.

Various scenarios were used for the simulation. For each scenario, 300 realizations of random variables were generated which represent 24 hour profiles. The realizations were obtained by the use of particular generators (described in the Section 2.1.4) on the forecasted profile. Realizations simulate inaccurate forecasts of weather conditions and cooling demand.

Figure 2.16 shows the example of one scenario which consists of the forecasted profiles of random variables (red lines). Particular realizations derived from the forecast profile are plotted by grey lines.

For each scenario, schedules were computed by all optimization methods – ADP using several approximation techniques, myopic optimization and full dynamic programming. Final schedules were ran on the forecasted profile as well as on all its derived realizations and average costs for given scenario were evaluated.

---

\[ ^7 \text{It solves Economic dispatch task (Subsection 2.1.4).} \]

\[ ^8 \text{This means various forecasts of the random variables.} \]
The experiments were conducted on the Intel Xeon 2.8 Ghz with 2 GB RAM, operating system Windows 7 (32 bit) and Matlab 2012b.

Table 2.5 summarizes the results achieved by particular optimization methods. The average cost column represents the cost of running the schedule on the forecasted realization averaged over all scenarios. The computing time column represents time needed for schedule computation.

Myopic strategy stands for the baseline for the assessment. In the cost cells, the number express absolute value of the operational costs and the number in the brackets means proportional costs to the baseline. Best results in terms of costs savings achieved by full dynamic programming are approx. 11%, but the computation time was significant. ADP algorithm achieved savings between 7-8% which is less than in the case of full dynamic programming. This is caused by using of approximated value of cost function $\hat{C}(f)$ instead of actual costs $C(x, u, \omega)$. On the other hand, use of cost function approximation reduces computational time significantly. For all alternatives of ADP, computational time is split into two terms. First term in the cell means the time spent on the initialization of approximation structure and second term stands for optimization time. Maximum and moving average aggregators spent less computational time than the RFWR aggregator. On the other hand, RFWR can approximate the space on similar accuracy level as other aggregators with lower memory requirements which are 130 kB for maximal aggregator, 480 kB for moving average aggregator and 35 kB for RFWR.

---

$^9$Initialization is performed just once and approximation architecture is not initialized on the next optimization.
### Table 2.5: Experimental Results

<table>
<thead>
<tr>
<th>Method</th>
<th>Avg. costs</th>
<th>Avg. computing time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Myopic</td>
<td>101.19 (100%)</td>
<td>1.7s</td>
</tr>
<tr>
<td>Dynamic programming</td>
<td>89.54 (88.49%)</td>
<td>2419s</td>
</tr>
<tr>
<td>Maximum</td>
<td>92.95 (91.86%)</td>
<td>252 + 391 = 643s</td>
</tr>
<tr>
<td>ADP Moving average</td>
<td>92.95 (91.86%)</td>
<td>231 + 370 = 601s</td>
</tr>
<tr>
<td>RFWR</td>
<td>93.49 (92.39%)</td>
<td>2407 + 1210 = 3617s</td>
</tr>
</tbody>
</table>

**Conclusion**

In this section, the microgrid energy management optimization problem was formulated and the MG model was briefly described. Further, optimization algorithm based on the approximate dynamic programming technique was introduced and several alternatives of the approximation architecture were presented. Hybrid state space vector, decision variables vector as well as feature vector were defined.

Described algorithm reduces computational time compared to full dynamic programming significantly (50-70% computational time reduction\(^{10}\)) while accuracy\(^{11}\) reduction caused by approximation is less than 5%. Three approximation architectures were introduced which offer a possibility of choice between memory requirements and computational performance of the algorithm.

Further research can be focused on analysis of another approximation architectures and value iteration approach to the ADP as extension of policy search.

### 2.2 Demand Response

This section is a summary of the work on modeling for demand side participation of thermostatically controlled loads, performed within the MoVeS consortium and presented in [54]. We propose three classes of models that approximate aggregate TCL dynamics. We analyze these models in terms of their accuracy and computational tractability. The models demonstrate a progression from models that help us analyze and predict TCL population behavior to those that help us develop large-scale automatic control strategies. Specifically, we demonstrate how formal methods from computer science and optimal control can be used to derive bounds on model error, guarantees for trajectory tracking, and algorithms for price arbitrage. We find that the accuracy of the analytic results decreases as TCL parameter heterogeneity is introduced. Thus, we motivate further development of analytical tools and modeling approaches to investigate realistic TCL behavior in power systems.

---

\(^{10}\)For the assessment, the optimization time was used.

\(^{11}\)The accuracy means deviation from results of full dynamic programming.

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2.2.1 Motivations

Household appliances such as water boilers/heaters, electric heaters, and air-conditioners, referred to as thermostatically controlled loads (TCLs) generally operate within a deadband around a temperature set point. These appliances can store energy due to their thermal mass. Thus, we can control them by turning them on/off prematurely or slightly adjusting their temperature set point, and still ensure that they can provide the service expected by the user. In this way, one can hope to control a large population of TCLs so that their aggregate power consumption tracks a signal or minimizes a cost. For example, the aggregate power can follow a desired trajectory determined based on the power fluctuations of renewable energy sources or the demand on the grid [28].

Control of the aggregate power output of a TCL population can provide a variety of benefits to the electricity grid. A key change to the grid is the increase in renewable energy sources, such as wind and solar, which are hard to accurately predict. This leads to the need for additional ancillary services [64, 45] such as control reserves which are today mainly provided by conventional generators. With Smart grid sensing and communication infrastructure, population of TCLs can provide additional means for providing control reserves [30]. There are several advantages to using TCLs for this task [29]. First, ancillary service needs can be partially addressed locally, which reduces the need for additional transmission line capacity. Second, using a large population of TCLs may improve robustness, since if a few TCLs fail to provide the required service, the consequence in the large population would be small. Third, the resource potential is large [73].

To integrate TCL aggregations into the power systems, several challenges in terms of modeling, control, and communication need to be addressed. First, although accurate models for individual TCLs exist, developing a computationally tractable and accurate aggregate model for a population of TCLs is challenging. Next, control schemes must be developed so that the population tracks a given power trajectory with a required accuracy or can minimize a cost function. The control must be robust with respect to uncertainties in the model and exogenous inputs such as weather, uncertainty in renewable energy forecasts, and uncertainty in price forecasts. Since an aggregator will perform the estimation and control of the TCL population, there must be a communication link between each TCL and the aggregator. The design of such communication infrastructure and trade-offs between its cost and accuracy in estimation and control is the subject of investigation [74].

This section proposes and examines several classes of models for analysis and control of TCL populations. Each proposed model makes certain assumptions about the TCL parameters and the information available to a central aggregator and has certain computational and deductive capabilities about the population performance. In particular, the objective of the first part is to derive an abstraction of the TCL population dynamics which analytically characterizes the process noise in the abstraction and derives bounds on the error between the power output of the TCL population and that predicted by the model. Partial results of this approach appeared in our recent work [92]. The additional contributions here include the consideration of a heterogeneous TCL population. The
second section explores formulation of a new method for modeling the uncertainties in TCL population. The objective of this section is to determine whether it is feasible to track a power signal within a required accuracy, given observations of the power consumption of the TCL population. The third section examines the potential of a TCL population to participate in energy markets through energy arbitrage. This section extends our recent results in [71] by finding upper bounds on energy savings, computed assuming each TCL optimizes its own energy costs. Each section works with a different model and all three models are simulated based on realistic parameter data for the TCLs. The tradeoffs in the accuracy, complexity, and application of the models are analyzed.

2.2.2 Background on TCL Modeling

Individual TCL Model

The starting point of all three models is the following discrete time Stochastic Hybrid System (SHS) [5], which describes the evolution of the temperature of a single cooling TCL [28, 49, 78, 100, 65]:

\[
\theta(t+1) = a \theta(t) + (1-a)(\theta_a - m(t)RP_{rate}) + w(t),
\]

where \(\theta\) is the temperature of the load, \(\theta_a\) is the ambient temperature, \(C\) and \(R\) indicate the thermal capacitance and resistance respectively, \(P_{rate}\) is the rate of energy transfer, and \(a = e^{-h/RC}\), with a discretization step \(h\). The process noise \(w(t) \in \mathbb{R}\) is independent identically distributed (i.i.d.) and characterized by a probability density function \(p_w\). The temperature dynamics are regulated by the discrete switching variable \(m(t) \in \{0, 1\}\).

The switching dynamics for a cooling TCL are

\[
m(t+1) = f(m(t), \theta(t)) = \begin{cases} 
0, & \theta < \theta_s - \delta/2 \\
1, & \theta > \theta_s + \delta/2 \\
m(t), & \text{else,}
\end{cases}
\]

where \(\theta_s\) denotes the temperature set-point and \(\delta\) the dead-band width. We define \(\theta_- = \theta_s - \delta/2\) and \(\theta_+ = \theta_s + \delta/2\) as the lower and upper boundary of the temperature range. The power consumption of a TCL at time \(t\) is equal to \(\frac{1}{\eta}m(t)P_{rate}\), where the parameter \(\eta\) is the Coefficient Of Performance (COP). For notation simplicity, we define

\[
\bar{P}_{rate} = \frac{1}{\eta}P_{rate}.
\]

In the discrete time model, the changes in the thermostat state occur only at discrete time steps.

Probability Distribution Modeling

One of the early works on TCL population models was [65]. Here, the authors focused on characterizing the probability distribution of homogenous populations of TCLs in temperature space. They derived partial differential equations (PDEs) for the evolution

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of the probability density function (PDF) of the TCLs in the On and Off mode. The PDF characterization suffers from a few drawbacks including computation of the solutions, extensions to non-homogeneous populations, and its use for control synthesis. An exact solution of the PDE was developed in [28]; however, it was shown that the aggregate power output predicted by the PDE does not approximate the entire temperature distribution accurately in the case of heterogenous loads.

In an effort to better understand TCL aggregate behavior, input-output models [28], state queuing models [59, 60] and Markov chain models have been proposed. Instead of using PDFs, the Markov chain models approximate the evolution of TCL populations over discretized temperature intervals and on/off modes, and at discrete time points [13, 56, 55, 72, 74]. The Markov chain models are in the class of linear time varying system

\[ X(t+1) = P^T X(t) + W(t), \]  

where \( X \) denotes a vector containing the probability mass of TCLs within each discretized temperature state and On/Off mode, \( P \) is the transition probability matrix, \( (\cdot)^T \) denotes the transpose of the matrix, and \( W \) denotes a noise term. This modeling approach was used for tracking control over short horizons and estimation of state. More detailed individual TCL models based on three state hybrid systems were used in [109] and a Markov chain abstraction of the population’s corresponding probability density function was also addressed.

In relation to past work, Section 2.2.3 provides insight into the Markov chain abstraction method (referred to as Model 1) by characterizing the process noise term \( W(t) \) of the aggregate model analytically and deriving bounds on the model error. Sections 2.2.4 and 2.2.5 present novel approaches for TCLs population modeling and are referred to as Model 2 and 3, respectively.

**Model Parameters**

We consider populations of air conditioners. Equations (2.30) and (2.31) are used as the plant. The parameters in Sections 2.2.3 and 2.2.4 are set according to Table 2.6 column 3 [28]. In the third modeling approach, in Section 2.2.5 a heterogenous population is considered and the parameters are uniformly distributed within the bounds given in Table 2.6 column 4 [72].

**2.2.3 Model 1 - A Markov Chain Abstraction with Analytic Error Bounds**

We develop a novel two-step abstraction procedure to derive a linear stochastic dynamical model for the TCL population. In the first step, a population of discrete time Markov chains is generated based on the probabilistic evolution of the continuous state model of each TCL temperature; in the second step, a Markov chain of reduced order is derived which is an exact representation of the population model, that is, it is probabilistically bisimilar to the original model. The approach is analytically developed for the case of a homogeneous population of TCLs, and extended to a heterogeneous population. While
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Model 1 &amp; 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_s$</td>
<td>set-point</td>
<td>20°C</td>
<td>18–27°C</td>
</tr>
<tr>
<td>$\delta$</td>
<td>dead-band width</td>
<td>0.5°C</td>
<td>0.25–1°C</td>
</tr>
<tr>
<td>$\theta_a$</td>
<td>ambient temperature</td>
<td>32°C</td>
<td>varying</td>
</tr>
<tr>
<td>$R$</td>
<td>thermal resistance</td>
<td>2°C/kW</td>
<td>1.5–2.5°C/kW</td>
</tr>
<tr>
<td>$C$</td>
<td>thermal capacitance</td>
<td>10kWh/°C</td>
<td>1.5–2.5 kWh/°C</td>
</tr>
<tr>
<td>$P_{rate}$</td>
<td>power</td>
<td>14 kW</td>
<td>10–18 kW</td>
</tr>
<tr>
<td>$\eta$</td>
<td>COP</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>$h$</td>
<td>time step</td>
<td>10 sec</td>
<td>10 sec</td>
</tr>
</tbody>
</table>

Table 2.6: Parameters for simulation of air conditioner populations from [28] and [72].

Throughout this section we use the notation $\mathbb{N}$ for natural numbers, $\mathbb{Z} = \mathbb{N} \cup \{0\}$, $\mathbb{N}_n = \{1, 2, 3, \cdots, n\}$, and $\mathbb{Z}_n = \mathbb{N}_n \cup \{0\}$. We denote vectors with bold typeset and with a letter corresponding to that of its elements.

### Abstraction of a Single TCL

The interpretation of (2.30)-(2.31) as a SHS enables the use of an abstraction technique first proposed in [2], aimed at reducing a discrete time, uncountable state space Markov process into a discrete time finite state Markov chain. This abstraction is based on state space partitioning as follows. Consider an arbitrary finite partition of the continuous domain $\mathbb{R} = \bigcup_{i=1}^n \Theta_i$, and arbitrary representative points within the partitioning regions denoted by $\{\bar{\theta}_i \in \Theta_i, i \in \mathbb{N}_n\}$. The hybrid state space is characterized by a variable $s = (m, \theta)$ in $\mathbb{Z}_1 \times \mathbb{R}$. Introduce a finite state Markov chain $\mathcal{M}$, characterized by $2^n$ states $s_{im} = (m, \bar{\theta}_i), m \in \mathbb{Z}_1, i \in \mathbb{N}_n$. The transition probability matrix related to $\mathcal{M}$ consists of the elements

$$P(s_{im}, s'_{im'}) = \delta_d[m' - f(m, \bar{\theta}_i)] \cdot \int_{\Theta_i'} p_\omega(\bar{\theta} - a \bar{\theta}_i - (1 - a)(\theta_a - mR_P))d\bar{\theta},$$

where $m' \in \mathbb{Z}_1, i' \in \mathbb{N}_n$ and $\delta_d[m' - f(m, \bar{\theta}_i)]$ is the Dirac-delta function. For ease of notation, we rename the states of $\mathcal{M}$ by the bijective map $\ell(s_{im}) = mn + i, m \in \mathbb{Z}_1, i \in \mathbb{N}_n$, and accordingly introduce the new notation

$$P_{ij} = P(\ell^{-1}(i), \ell^{-1}(j)), \quad \forall i, j \in \mathbb{N}_n.$$  

Due to the presence of switching dynamics, the conditional density function of the stochastic system describing the dynamics of a single TCL is discontinuous. The selection of the partitioning sets then requires special attention. It is convenient to select a
partition for the dead-band $[\theta_-, \theta_+]$, thereafter extending it to a partition over the whole line $\mathbb{R}$ as shown in Figure 2.17. Let us select two constants $l, m \in \mathbb{N}$, $l < m$, compute the partition size $\tau = \delta/2l$ and quantity $L = 2m\tau$. Now construct the boundary points of the partition sets $\{\theta_i\}_{i=-m}^m$ for the temperature axis as

$$\theta_{\pm 1} = \theta_s \pm \delta/2, \quad \theta_{\pm m} = \theta_s \pm L/2, \quad \theta_{i+1} = \theta_i + \tau,$$

$$\mathbb{R} = \bigcup_{i=1}^n \Theta_i, \quad \Theta_1 = (-\infty, \theta_-), \quad \Theta_n = (\theta_m, \infty),$$

$$\Theta_{i+1} = (\theta_{-m+i-1}, \theta_{-m+i+1}), \quad i \in \mathbb{N}_{n-2}, \quad n = 2m + 2.$$

We render the Markov states of the infinite length intervals $\Theta_1, \Theta_n$ absorbing, that is, once the temperature reaches one of these intervals, it remains there forever.

**Abstraction of a Homogeneous Population of TCLs**

Consider now a population of $n_p$ homogeneous TCLs, that is a population of TCLs which, after possible rescaling of (2.30)-(2.31), share the same set of parameters $\theta_s, \delta, \theta_a, R, C, P_{rate}, \eta, h$, and the distribution $p_w$ of $w(t)$. Each TCL can be abstracted as a Markov chain with the transition probability matrix $P = [P_{ij}]$, where $i, j \in \mathbb{N}_{2n}$. This abstraction leads to $n_p$ identical Markov chains $M$.

The homogeneous TCL population can be represented by a single Markov chain $\Xi$, built as the cross product of the $n_p$ Markov chains. The state of the Markov chain $\Xi$ is

$$z = [z_1, z_2, \cdots, z_{n_p}]^T \in \mathcal{Z} = \mathbb{N}_{2n}^{n_p},$$

where $z_j \in \mathbb{N}_{2n}$ represents the state of the $j^{th}$ Markov chain. We denote by $P_\Xi$ the transition probability matrix of $\Xi$.

The Markov chain $\Xi$ has $(2n)^{n_p}$ states, which in general can be very large. As the second step of the abstraction procedure, we are interested in further aggregating this model. The motivation for this approach stems from the fact that for studying aggregate power consumption, it is sufficient to know the number of TCLs in each discrete state. Formally, the aggregation is achieved through the notion of (exact) probabilistic bisimulation [12].

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Let us introduce $AP$ as a constrained vector with a dimension corresponding to the number of states of Markov chain $M$:

$$AP = \left\{ x = [x_1, x_2, \ldots, x_{2n}]^T \in \mathbb{Z}_p^{2n} \mid \sum_{i=1}^{2n} x_i = n_p \right\}.$$ 

A function $L : \Xi \to AP$, referred to as a labeling function, associates to a configuration $z$ of $\Xi$ a vector $x = L(z)$, the elements $x_i \in \mathbb{Z}_{np}$ of which count the number of TCLs in bin $i$, $i \in \mathbb{N}_{2n}$. Notice that the set $AP$ is finite with cardinality $|AP| = (n_p + 2n - 1)!/(n_p!(2n-1)!)$, which for $n_p \geq 2$ is much less than the cardinality $(2n)^{np}$ of $\Xi$. Define an equivalence relation $R$ on the state space of $\Xi$, such that

$$\forall(z, z') \in R \Leftrightarrow L(z) = L(z').$$

This equivalence relation provides a partition of the state space of $\Xi$ into equivalence classes belonging to the quotient set $\Xi/R$, where each class is uniquely specified by the label of its elements. The equivalence relation $R$ is an exact probabilistic bisimulation relation on $\Xi$ [12], which means for any set $T \in \Xi/R$

$$P_{\Xi}(z, T) = P_{\Xi}(z', T),$$

where $P_{\Xi}(z, T) = \sum_{z_1 \in T} P_{\Xi}(z, z_1)$. Given an observation $x(t) \in AP$ at time $t$ over the Markov chain $\Xi$, it is of interest to compute the probability mass function of the conditional random variable $(x_i(t+1)|x(t))$ as $P(x_i(t+1) = k|x(t))$, for any $k \in \mathbb{Z}_{np}, i \in \mathbb{N}_{2n}$. Using the law of total probability we get the following result.

**Theorem 1** The conditional random variables $(x_i(t+1)|x(t))$ have Poisson-binomial distributions, whereas the conditional random vector $(x(t+1)|x(t))$ has a generalized multinomial distribution [17]. Their mean, variance, and covariance are characterized by

$$\mathbb{E}[x_i(t+1)|x(t)] = \sum_{r=1}^{2n} x_r(t) P_{ri},$$

$$\text{var}(x_i(t+1)|x(t)) = \sum_{r=1}^{2n} x_r(t) P_{ri}(1 - P_{ri}),$$

$$\text{cov}(x_i(t+1), x_j(t+1)|x(t)) = -\sum_{r=1}^{2n} x_r(t) P_{ri} P_{rj},$$

for all $i, j \in \mathbb{N}_{2n}, i \neq j$.

Theorem [1] indicates that the distribution of the conditional random variable $(x(t+1)|x(t))$ is independent of the underlying state $z$ of $\Xi$ in which $L(z) = x$.

Without loss of generality, let us normalize the values of the labels $x$ by the total population size $n_p$, thus obtaining a new variable $X$. Based on the expression of the first two moments of $(X(t+1)|X(t))$, we apply a translation (shift) on this conditional random vector which allows expressing the following dynamical model for the variable $X$:

$$X(t+1) = P^T X(t) + W(t),$$

(2.34)

where the distribution of $W(t)$ depends only on the state $X(t)$. We use the Lyapunov central limit theorem [17] to show that this distribution converges to a Gaussian one.
**Theorem 2** The random variable \( (X_i(t+1)|X(t)) \) can be explicitly expressed as

\[
X_i(t+1) = \sum_{r=1}^{2n} X_r(t)P_{ri} + \omega_i(t),
\]

where the random variables \( \omega_i(t) \) converge (in distribution) as \( n_p \to \infty \) to the Gaussian random variables \( \omega_i(t) \sim \mathcal{N}(0, \sigma^2_i(X(t))) \), \( \sigma^2_i(X) = \frac{1}{n_p} \sum_{r=1}^{2n} X_r P_{ri}(1 - P_{ri}) \).

We have modeled the evolution of the TCL population with an abstract model based on linear stochastic difference equations (2.34). The approach in derivation of the stochastic model above is different than that of [74] in that the noise covariance is derived analytically in the current approach, in contrast to estimated via simulation in the latter approach.

**Quantification of the Abstraction Error**

The total power consumption obtained from the aggregation of the individual models in (2.30)-(2.31), with variables \( (m_i, \theta_i)(t), i \in \mathbb{N}_{n_p} \), denoting TCL \( i \) is given as

\[
y_{total}(t) = \sum_{i=1}^{n_p} m_i(t)\bar{P}_{rate}.
\]

Focusing on the abstract model, described in terms of the normalized variable \( \bar{X} \), the power consumption is equal to

\[
y_a(t) = H\bar{X}(t), \quad H = \bar{P}_{rate}[0_n, 1_n],
\]

where \( 0_n, 1_n \) are n-dimensional row vectors with entries equal to zero and one, respectively. The following theorem quantifies the abstraction error over the total power consumption.

**Theorem 3** Consider a homogeneous population of TCLs with a Gaussian process noise \( w(\cdot) \sim \mathcal{N}(0, \sigma^2) \), and the abstracted model constructed based on the partitions defined in (2.33). The difference in the expected value of the total power consumption of the population \( y_{total}(t) \), and that of the abstracted model \( y_a(t) \), both conditional on the corresponding initial conditions, is upper bounded by

\[
\left| \mathbb{E}[y_{total}(t)|s_0] - \mathbb{E}[y_a(t)|X_0] \right| 
\leq n_p(t-1)\bar{P}_{rate} \left[ \frac{(t-2)}{2} \epsilon + \frac{2a}{\sigma\sqrt{2\pi}} \right], \tag{2.35}
\]

where the constants above are given as

\[
\epsilon = \frac{e^{-\gamma^2/2}}{\gamma\sqrt{2\pi}},
\]

\[
\gamma = \frac{1 - a}{2\sigma} \left[ \frac{L^t + \delta}{1 - a^t} - R\bar{P}_{rate} - |2(\theta_s - \theta_a) + R\bar{P}_{rate}| \right],
\]

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for all $s_0 \in (\mathbb{Z}_1 \times [\theta_{-m}, \theta_m])^n$. The initial state $X_0$ is a function of the initial states in the population of TCLs $s_0$, according to the definition of the state vector $X$.

The importance of this theorem is that it allows us to tune the error in estimating the total power consumption of the population from the abstraction. Notice that the constant $\gamma$ is an affine function of $L$. The constant $\epsilon$, and consequently the first term of the error bound (2.35), is reduced by selecting a larger interval around the dead-band to be partitioned. The second term of the error bound (2.35) is decreased by a smaller partition diameter $\tau$. The error bound depends linearly on the population size since the total power consumption is the sum of power consumption of $n_p$ single TCL. The error bound depends quadratically on time. Currently, we are exploring improvements to this bound.

**Extension to a Heterogeneous Population of TCLs**

Consider a heterogeneous population of $n_p$ TCLs, where heterogeneity is characterized by a parameter $\alpha$ that takes $n_p$ values. Each instance of $\alpha$ specifies a set of model parameters $(\theta_s, \delta, \theta_a, R, C, \sigma)$ for a single TCL. The dynamical model can be abstracted as a Markov chain $M_\alpha$ with a transition matrix $P_\alpha = [P_{ij}(\alpha)]_{i,j}$. This transition probability matrix obtained for a TCL depends on its own set of parameters specified by $\alpha$. The apparent difficulty is that the heterogeneity in the transition probability matrices renders the quantity $P(x_i(t+1) = j|x(t))$ dependent not only on the label $x(t) = L(z(t))$, but also on the current state $z(t)$.

In contrast to the homogeneous case, which allows us to compute probabilities $P(x_i(t+1) = j|x(t))$ by constructing an exact probabilistic bisimulation of $\Xi$, in the heterogeneous case we have to leverage approximate probabilistic bisimulation of the Markov chain $\Xi$. In this case, the approximation error can only be quantified empirically using the matrix $P(z(t+1)|z(t))$, which in practice can be unfeasible. We encompass the population heterogeneity by constructing an empirical probability distribution $f_\alpha(\cdot)$ from the finite set of values for parameter $\alpha$.

**Theorem 4** If the TCL population heterogeneity is characterized by a parameter $\alpha$ with empirical distribution $f_\alpha(\cdot)$, using the approximate probabilistic bisimulation of the Markov chain $\Xi$, the random vector $(X(t+1)|X(t))$ has the expected value $M(X(t)) = P^T X(t)$ and covariance matrix $\Sigma(X(t))$, where for all $i, j \in \mathbb{N}_{2n}, i \neq j$,

$$
\Sigma_{ij}(X) = \frac{1}{n_p} \sum_{r=1}^{2n} X_r P_{ri}(1 - P_{ri}) \\
+ \frac{1}{n_p - 1} \left( \sum_{r=1}^{2n} X_r P_{ri}^2 \right)^2 - \frac{1}{n_p - 1} \sum_{r=1}^{2n} X_r P_{ri}^2,
$$
\[ \Sigma_{ij}(X) = \frac{1}{n_p - 1} \left( \sum_{r=1}^{2n} X_r P_{ri} \right) \left( \sum_{s=1}^{2n} X_s P_{sj} \right) 
- \frac{1}{n_p - 1} \sum_{r=1}^{2n} X_r P_{ri} P_{rj} 
- \frac{1}{n_p - 1} \sum_{r=1}^{2n} X_r P_{ri} P_{rj}. \]

The bar notation indicates the expected value with respect to the parameters set \( \alpha \), for instance, 
\[ P_{ri} P_{rj} = \mathbb{E}_\alpha [P_{ri}(\alpha) P_{rj}(\alpha)] = \int P_{ri}(v) P_{rj}(v) f_\alpha(v) \, dv. \]

Theorem 4 enables us to use model (2.34) as an approximation of the dynamics of the heterogeneous population. The transition matrix \( P \) and the covariance of \( W(t) \) must be computed with respect to the set of parameters, according to the Theorem 4.

**Numerical Benchmark**

In this section, we compare the performance of our formal abstraction with a deterministic abstraction. A TCL population size of \( n_p = 500 \) is considered for all the simulations. Each TCL is characterized by parameters that take values in Table 2.6. All TCLs are initialized in the Off mode \( (m(0) = 0) \) and with a temperature at the set-point \( (\theta(0) = \theta_s) \). We assume the process noise has a Gaussian distribution with a standard deviation \( \sigma = 0.01 \sqrt{h} = 0.032 \).

For the formal abstraction proposed in this work, we construct a partition as per (2.33) with \( l = 7 \), \( m = 35 \), which leads to \( 2n = 144 \) abstract states. We then generate the abstracted system trajectory using Equation (2.34), where the covariance matrix of the noise term \( W(t) \) is that given by the limiting covariance in Theorem 2. We run 50 Monte Carlo simulations for the TCL population based on the explicitly aggregated dynamics in (2.30)-(2.31) and compute the average total power consumption.

For comparison, we also perform a deterministic abstraction which does not consider the analytically derived covariance matrix for the process noise \( W(t) \). We select a \( n_d = 5 \) for the number of bins for this abstraction, which leads to 10 states. This selection is based on empirical tuning targeted toward optimal performance – however, there seems to be no clear correspondence between the choice of \( n_d \) and the overall precision [55]. Figure 2.18 (top) presents the results of the experiment. As can be observed, generating the stochastic model based on (2.34) results in more accuracy than a deterministic abstraction.

To observe the performance of the proposed model for the heterogeneous population, let us assume that heterogeneity enters the TCL population in only the thermal capacitance \( C \) of each single TCL, which is taken to be \( C \sim U([2, 18]) \), that is a uniform distribution over a compact interval. We perform 50 Monte Carlo simulations with a noise level \( \sigma = 0.032 \), and select discretization parameters \( n_d = 7, l = 10, \) and \( m = 50 \). The outcome is presented in Figure 2.18 (bottom).
Control of TCL Population

Among the different strategies for controlling the total power consumption of a population of TCLs, we consider the case in which the control input is the set-point $\theta_s$ of the TCL [28]. We intend to apply the same control input to all TCLs since this requires no prior knowledge of the state of the single TCL. Given the model parameters, we use online measurement of the total power consumption of the TCL population, to estimate the states in $X(t)$ and we use the set-point $\theta_s$ to track any reference signal based on a one-step output prediction.

Suppose we have a homogeneous population of TCLs with known parameters. Based on Equation (2.34), we set up the model

$$X(t + 1) = P^T(\theta_s(t))X(t) + W(t),$$

where $\theta_s(t)$, the set-point value at time $t$, is the control input for the model. We assume
Figure 2.19: Tracking a piecewise constant reference signal (top) by set-point control (bottom) in a homogeneous population of TCLs.

that the control input is discrete and take values from a set:

$$\theta_s(t) \in \{\theta_{-1}, \theta_{-1+1}, \cdots, \theta_{t-1}, \theta_t\}, \quad \forall t \in \mathbb{Z}.$$ 

This assumption makes it possible to use the partitions defined in Equation set (2.33) at all time steps. The process noise $W(t)$ is normal with zero mean and its state-dependent covariance matrix is obtained from Theorem. The total power consumption of the TCL population is measured as $y_m(t) = HX(t) + \nu(t)$, where $\nu(t) \sim \mathcal{N}(0, R_v)$ is a measurement noise and $\sqrt{R_v}$ represents a standard deviation which depends on the real-time measurements from power meters. Since the process noise $W(t)$ is state-dependent, the state of the system can be estimated by modifying the classical Kalman filter.

Once the state estimates are available, the following one-step Model Predictive Control
scheme is employed to synthesize the control input in the next step:

$$\min_{\theta_s(t)} |\hat{y}(t+1) - y_d(t+1)|$$

s.t. $\hat{X}(t+1) = F(\theta_s(t))\hat{X}(t)$

$$\hat{y}(t+1) = H\hat{X}(t+1)$$

$$\theta_s(t) \in \{\theta_{l-1}, \theta_{l-1+1}, \ldots, \theta_{l-1}, \theta_l\},$$

where $y_d(\cdot)$ is the reference signal and $\hat{X}(t)$ is the state estimate provided by Kalman filter. The obtained optimal value for $\theta_s(t)$ is applied to the TCL population at the following iteration.

This scheme is implemented on a homogeneous population of $n_p = 500$ TCLs, for tracking a randomly generated piecewise constant reference signal. We set the discretization parameters to $l = 8$, $m = 40$, where the standard deviation of the measurement noise is $\sqrt{\sigma_v} = 0.005$. Figure 2.19 displays the tracking outcome (top), as well as the required set-point signal (bottom) synthesized from the above optimization problem.

### 2.2.4 Model 2 - Constraint Satisfaction Formulated via Satisfiability Modulo Theory

Here, we explore a different method for capturing the uncertainty in an individual TCL’s temperature evolution and in bounding the aggregate power consumption of the TCL over a future time horizon given an input sequence. We quantify this bound by solving a feasibility problem whose constraints are determined from the uncertain dynamics of the TCLs and the historical data on population power consumption. Our results are preliminary. We show that the approach works well for deterministic systems but more work is needed to handle stochastic systems and heterogeneous parameters.

The problem of interest here is to construct a controller that comes with guaranteed performance. In particular, given a sequence of observed power consumption and a known sequence of control inputs, we seek to choose an input to apply in the next time frame, such that the resulting overall power will be guaranteed to lie in an interval $[P^* - \epsilon, P^* + \epsilon]$ around a desired power $P^*$. Instead of developing an abstraction model and then quantifying its resulting error, we use observations of the population to formulate meaningful constraints on the future trajectory of the population given a control input.

We derive a model that is a continuous time abstraction, with the temperature interval of the TCL discretized into bins. We then track the movement of the upper and lower boundaries of each bin. The continuous (temperature) and discrete (On/Off mode) states of the bin boundary provide constraints on the continuous and discrete state of a TCL whose initial temperature is within the bin. To cope with the continuous dynamics and the uncertainty in the states of individual TCLs in a given bin, we use the formalism of the satisfiability modulo theory (SMT). An SMT instance is a formula in first-order logic, and the problem is determining whether such a formula is satisfiable. SMT is a widely used method in computer science verification, and solvers such as iSAT [39] have been developed to automatically verify and analyze model properties given a set
of initial conditions or control strategies. Using an SMT solver, we provide upper and lower bounds on the performances of a set of control strategies.

**Model Description**

We use the following continuous time dynamics of the TCL, which can be derived by letting $h \to 0$ from (2.30) and by removing the noise term $w$:

\[
\frac{d\theta(t)}{dt} = \frac{1}{R(t)C} (\theta_a - m(t)R(t)P_{rate} - \theta(t))dt
\]

\[
dm(t) = \begin{cases} 
-1, & \theta(t) < \theta_- + u(t) \\
1, & \theta(t) > \theta_+ + u(t) \\
0, & \text{otherwise}
\end{cases}
\]

\[R(t) = \begin{cases} 
R_0, & N(t) \equiv 0 \mod 2 \\
R_1, & N(t) \equiv 1 \mod 2
\end{cases}
\]  

(2.36)

In the above, $u(t)$ is the control input, $R(t) \in \{R_0, R_1\}$ capture two different thermal resistance values (reflecting, for example, closed vs. open windows); and the switching times between the resistance values are distributed according to the homogeneous Poisson process $N_t$ with a specified rate $\lambda$. The corresponding dynamics are illustrated for a single household by a sample path in the left panel of Figure 2.20. Although we develop the approach for thermal resistances taking two potential values, the approach could be generalized to multiple values of thermal resistances, to account for various discrete changes in the room such as occupancy or opening/closing of entrances.

The reasoning for modeling stochasticity in this way is twofold. First, changes in thermal resistance due to changes in room occupancy, opening and closing of doors/windows may be more accurate than fluctuations in the temperature captured by the noise term used in Equation (2.30). Second, modeling the random influences as jumps in the thermal resistances allows for an event based simulation. Although the underlying model is formulated in continuous time, only discrete events have to be considered. Also, between two random events, the dynamics are deterministic. Therefore, we can draw exact samples from the continuous time model.

**Abstraction Approach**

Given that some of the parameters and the control strategy are not known beforehand, obtaining guaranteed bounds on the power consumption of the TCL population in future time steps requires solving a hard optimization problem. To reduce the computational complexity we aim at a safe abstraction of the model in (2.36) as follows:

1. We divide the temperature interval into bins and count the number of TCLs within a bin. The bins are allowed to move along the temperature axis over time, in contrast to the previous approaches.
2. By introducing non-determinism to capture the unknown temperature of individual TCLs within a bin and uncertainty in switching dynamics of the thermal resistance $R(t)$ we safely over-approximate the effects of these variables.

Here, we are only interested in properties of the power consumption of the population. Thus, we adopt an event based time resolution within the abstraction approach by considering points in time at which the power can potentially change.

**Abstraction dynamics** Mathematically, the dynamics of the model can be formulated as follows: Each bin $i$ is characterized by a tuple $(\theta^u_i, \theta^l_i, m^u_i, m^l_i, R_i)$ denoting the temperature of its upper boundary $\theta^u_i$, its lower boundary $\theta^l_i$ (indicated in Fig. 2.20 right panel, with blue lines). Each has an associated On/Off state $m^u_i, m^l_i$ and a resistance $R_i \in \{R_0, R_1\}$. A bin is defined for every combination of temperature range, discrete state $m^l = m^u \in \{0,1\}$ and $R \in \{R_0, R_1\}$. The total number of bins is therefore given by $4 \cdot n_d$, where $n_d$ denotes the number of bins along the temperature axis. Initially, all TCLs within bin $i$ have a temperature between $\theta^u_i$ and $\theta^l_i$, start with a thermal resistance $R_i$ and are all in the same On/Off state, $m^u_i = m^l_i$.

The temporal evolution of the bounds of all bins can be calculated using the noiseless version of (2.36) and hence we can compute the first time any of the temperature bins hits one of the dead-band boundaries. These switching thresholds, $\theta_-$ and $\theta_+$, are denoted in Fig. 2.20 by black horizontal lines. Once one of the bin boundaries hits the threshold, the discrete state associated with the upper temperature bound $m^u_i$, and the one for the lower temperature bound $m^l_i$ start to differ indicating that not all TCLs within this bin need to have the same On/Off state. To indicate the temporal dependence of the $m$-values, we write $m^u_i(t_j), m^l_i(t_j)$ and $\theta^u_i(t_j), \theta^l_i(t_j)$ respectively.

The bin description above gives an over-approximation of the dynamical model in (2.36). That is, one can construct a sequence of On/Off states $\tilde{m}(t_j)$ which always fulfill $\min(m^u(t_j), m^l(t_j)) \leq \tilde{m}(t_j) \leq \max(m^u(t_j), m^l(t_j))$ but for which one cannot find a
temperature sequence counterpart $\tilde{\theta}(t_j), \tilde{m}(t_j)$ which fulfills the dynamical constraints at the same time. On the other hand, each trajectory $\tilde{\theta}(t_j), \tilde{m}(t_j)$ fulfilling the dynamical constraints, will also fulfill $\min(m^u(t_j), m^l(t_j)) \leq \tilde{m}(t_j) \leq \max(m^u(t_j), m^l(t_j))$ for some $i$. Therefore, the abstraction gives a safe over-approximation of the system in (2.36).

If there are more than one bins for which $m^l(t_j) \neq m^u(t_j)$, we cannot determine exactly how many TCLs fall into these bins based on observations of the overall power consumption. Nevertheless, these variables define upper and lower bound constraints on the number of TCLs within each bin. Within a SAT-based approach this non-determinism has to be resolved by a solver which can decide how many TCLs to put in a bin to fulfill all constraints.

The noise process can also be tackled by introducing non-determinism. To this end, we can calculate the temporal evolution of each bin given its initial condition (represented by the tuple) assuming no noise. Given two bins with different initial $R$ values but potentially the same On/Off state, one can now check if the evolutions cross or overlap each other in the temperature axis. If so, it is possible for a TCL within one bin to jump to the other bin by changing its $R$ parameter. This procedure leads to a set of bins $M_i$ to which a TCL within bin $i$ can jump to due a change in its dynamics, in this case the resistance $R$. Computing this for all time $t_j$ leads to a sequence of sets $M_i(t_j)$ which indicate the possibility of a jump within the time-frame $[t_j, t_{j+1}]$.

The computational load is increased by introducing non-determinism, that is, the solver has to decide how many TCLs to put in each bin based on the available constraints. However, the effort for handling multiple TCLs is drastically decreased since only the number of TCLs within a bin are determined.

**Controller Verification Using the SMT framework**

We consider a discrete time controller that acts on all TCLs by setting the same temperature set point $\theta_s$. For every given sequence of such a control signal, the quantities describing the bins can be calculated beforehand, which characterizes the behavior of a population of TCLs. The observed sequence of population power $P(t_k)$, can then be used to write down a set of constraints on the number of TCLs $n_i(t_k)$ within a bin $i$ at a given point in time $t_k$. More precisely, as we have upper and lower bounds $m^u_i(t_k), m^l_i(t_k)$, on the state of the TCLs within a given bin $i$, we have the following constraints on $n_i(t_k)$:

$$\sum_i m^l_i(t_k)n_i(t_k) \leq P(t_k) \leq \sum_i m^u_i(t_k)n_i(t_k) \tag{2.37}$$

The number of TCLs within a bin changes over time as the individual TCLs may change their dynamics due to the probabilistic switches in the $R$ parameters. To capture this behavior in the constraints, we introduce $Q(i, j, t_k)$ to keep track of the number of TCLs jumping from bin $i$ to bin $j$ within the time-interval $[t_{k-1}, t_k]$. Thus, in addition
to Constraint (2.37) we have the following two constraints:

\[ n_j(t_k) = \sum_i Q(i, j, t_k) \]  \hspace{1cm} (2.38a)

\[ Q(i, j, t_k) \leq \sum_{i'} Q(i', i, t_{k-1}) M(i, j, t_k), \]  \hspace{1cm} (2.38b)

where \( M(i, j, t_k) \in \{0, 1\} \) is a pre-computed deterministic quantity, which indicates whether it is possible for a TCL in bin \( i \) to jump to bin \( j \) within \([t_{k-1}, t_k]\) according to the noisy dynamics. Additionally, as there are \( n_p \) number of TCLs distributed across the bins, we have the trivial constraint:

\[ \sum_{i,j} Q(i, j, t_k) = n_p. \]  \hspace{1cm} (2.39)

All together, we can construct a set of constraints that give a safe over-approximation of the behavior of a population of TCLs, given an observed sequence of aggregate power consumption and a sequence of applied control inputs. To verify a controller, we check the feasibility of the constraints (2.37)-(2.39), with the following additional constraints capturing the performance requirements in the time step \( t_k \):

\[ P^u(t_k) := \sum_j m_j^u(t_k)n_j(t_k) \leq P^* + \epsilon, \]  \hspace{1cm} (2.40a)

\[ P^l(t_k) := \sum_j m_j^l(t_k)n_j(t_k) \geq P^* - \epsilon. \]  \hspace{1cm} (2.40b)

If all constraints are feasible, then the given control input is guaranteed to produce an aggregate power which is within \( \epsilon \) bound of \( P^*(t_k) \), the desired power at the next time step.

Under all control inputs which satisfy the performance guarantees, we can then either choose randomly or optimize with respect to further objectives. Note that the constraints do not consider the probability of the set of trajectories fulfilling the constraints but only characterize an over-approximation (superset) of the set of trajectories. Therefore, rare extreme situations are covered at the cost of potentially pessimistic guarantees.

**Simulation of the Abstraction Procedure**

To illustrate the approach, we simulated 200 TCLs. All parameters used for the simulation are summarized in Table 2.6. The number of bins was set to 22. In Figure 2.21 upper panel the individual temperature trajectories as well as the control inputs in terms of the desired temperature set-point is shown. We observe the population power, shown in the lower panel of Figure 2.21 for over18 time steps and use this information to construct a set of constraints for the next 3 minutes, as described above.

Figure 2.21 shows the result for the deterministic setting, that is \( R = 2.0 \) and is not changing. Given a set of control inputs, we determine which of them satisfy aggregate power bounds and if they satisfy a given bound \( \epsilon \), what is the smallest bound \( \epsilon' \) they

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can satisfy. We then implemented a controller that guarantees the least bound, that is least tracking error. The resulting guaranteed range for the chosen control inputs over the time horizon is shown as a gray region in the lower panel.

In principle, the approach can capture heterogeneity in parameters and stochasticity, e.g. switching \( R \) values. In this case, the worst and best case evolution of the bin temperature boundaries lead to temperature ranges and discrete states of each bin upper and lower boundaries which drift apart from each other over time. As a result, the guaranteed predictions are pessimistic. Because of too much conservatism, this approach does not work well for the heterogenous or stochastic models. To address this conservatism, one can use more bins, resulting in increased computational load. In particular, the approach scales linearly in terms of the observed history length and scales quadratically in terms of the number of bins.
2.2.5 Model 3 - TCL Population Model for Analyzing Arbitrage Potential

The objective of this section is to understand if non-disruptive direct load control of TCL aggregations could be used to arbitrage intra-hour electricity market prices. Given that price variability is on the orders of minutes, to address this problem we need models that allow us to optimize the power consumption of TCL aggregations over a horizon of minutes to hours. Most models, for example, the Markov chain model, were developed for short prediction horizons during which the ambient temperature remains approximately constant. Here, we need to account for time-varying temperature and longer prediction horizons. The results in this section are an extension of the work presented in [71].

The energy arbitrage problem has been investigated by a number of researchers [101, 104, 37, 91, 66, 98]. However, past research has not taken into account the specific capabilities and constraints of TCL aggregations. Recently, we developed an aggregate model of a heterogeneous TCL population and used this model to derive practically-feasible upper bounds on the amount of money that TCL aggregations could save through energy arbitrage in 5-minute energy markets [71]. We assumed that an aggregator rather than individual TCLs, arbitrages prices, and that the aggregator sends control signals to individual TCLs based on aggregate models and aggregate measurements only; he does not have access to individual TCL parameters or states. Here, we extend our analysis by comparing our previous results to those generated for the case when each individual TCLs arbitrages prices. This gives us actual upper bounds for the TCL energy arbitrage problem and helps us understand the value of information. In both cases, we assume control is via on/off switching, not temperature set point adjustment. Moreover, we assume that TCLs can only be controlled when they are within their dead-band, which ensures that our control is non-disruptive to the end users.

Arbitrage for an Individual TCL

We consider the case in which each TCL optimizes its power consumption given a forecasted price signal. Consider the individual TCL model (2.30). We define the control for each TCL to be $u \in \{0, 1\}$, where 0 turns a TCL off and 1 turns a TCL on. The discrete variable $m$ is now updated as follows:

$$
m(t+1) = \begin{cases} 
0, & \theta(t+1) < \theta_- \text{ or } \\
& u(t) = 0 \wedge \theta(t+1) \in [\theta_- , \theta_+] \\
1, & \theta(t+1) > \theta_+ \text{ or } \\
& u(t) = 1 \wedge \theta(t+1) \in [\theta_- , \theta_+] \\
& m(t), \text{ otherwise}
\end{cases}
$$

(2.41)
Let \( l(t) \) be the cost of energy at time step \( t \) and \( N \) be the prediction horizon. The arbitrage problem for one TCL can be written as:

\[
\min_{u \in U} \sum_{t=t_0}^{t_0+N} l(t)m(t)\bar{P}_{rate}
\]

\[
\text{s.t. } (2.30) \text{ and } (2.41),
\]

where \( U = \{0, 1\}^N \) and as a reminder \( \bar{P}_{rate} = P_{rate}/\eta \).

The above optimization problem can be solved using Dynamic Programming (DP). We tackle the DP problem by discretizing the state space. Each state represents both a specific temperature interval within or just outside the dead-band and the On/Off state. We then precompute all possible state transitions for each input and a range of outdoor air temperatures. With this information we can compute the optimal policy given forecasts for outdoor air temperatures and electricity prices.

In general, the cost of each state at time \( t \) is \( m(t-1)\bar{P}_{rate} \) since a TCL switches only at the end of each time step. To formulate the DP, we need to assign a cost to each state that is not a function of time. The cost of states within the dead-band is simply \( m\bar{P}_{rate} \); however, the cost of states just outside of the dead-band is a function of whether the TCL has just switched or not. Here, we assume that if a TCL is outside of the dead-band, it has just switched and its cost is \( (1-m)\bar{P}_{rate} \). We can choose the discretization step so that this is true nearly all of the time. However, sometimes this may not be the case. To solve this problem, one could introduce virtual states that capture not only temperature and on/Off state but recent switching history, but we leave this to future work.

An important consideration when picking the discretization step is that TCLs move at different speeds at different outdoor air temperatures. Therefore, a good discretization step for a high temperature may not be good at relatively low temperatures. For example, if the discretization step is too large and the outdoor air temperature is just above the dead-band then TCLs in the Off state may move so slowly that they do not switch bins in each time step. Decreasing the discretization steps alleviates this problem but can lead to others including numerical issues and divergence from the DP cost approximation described above. In sum, the DP works much better if the outdoor temperature is well above the dead-band temperatures.

### Results of Individual Optimization

We consider a 10 hour period with highly volatile prices (from California ISO node MERCEDE_1_N001 [26]) and high outdoor air temperature (from NOAA weather station Merced 23 WSW [80]). We use a population of \( n_p = 1,000 \) central air conditioners parameterized with the heterogeneous parameters in Table 2.6 and we assume \( w(t) = 0 \forall t \) in order to get an upper bound on the savings. For each TCL, we divide its dead-band into 100 temperature intervals and so we end up with 200 within-dead-band states, and we use the same discretization step for states just outside the dead-band. We precompute all bin transitions for only integer values of temperatures and use this as a
look-up table. We compute the optimal control policy based on the discretized system and perfect price and temperature forecasts, and apply that to the TCL model of (2.30) and (2.41). Figure 2.22 shows an example uncontrolled and controlled trajectory for one TCL. We find that the population saves about 28% of its total energy costs, while individual TCLs save -15% to over 55% of their individual energy costs (Figure 2.23). Differences in savings results from different thermal parameters, initial conditions, and DP model accuracy.

Since we have assumed a deterministic system and perfect forecasts, the analysis here provides upper bounds on energy cost savings through arbitrage for individual TCLs. In reality, the individual TCLs may not have direct access to time-varying price signals or may not have the local computational capabilities to do the optimization. Therefore, we consider a more realistic scenario in which a load aggregator uses aggregate models, aggregate measurements, and forecasts to compute optimal control trajectories and then

Figure 2.22: Example of individual TCL arbitrage results.
coordinate control responses by sending broadcast control signals to TCLs.

**Aggregate Thermal Battery Model**

For a population of TCLs, the previous optimization problem can be solved by formulating the cost function as the aggregate of costs of the individual TCLs: 

\[ h \sum_{t_{0}+N}^{t_{0}} \sum_{t_{i}=1}^{N_{p}} l(t) m(t) \bar{P}_{\text{rate}}. \]

This approach is impractical for two reasons. First, the aggregator would need to know all of the individual TCL parameters and states. Second, the aggregator would need communication links with each individual TCL to send individual on/off control signals. Here we assume that the aggregator only has access to aggregate system parameters and measurements and that he can only broadcast control vectors to TCL populations, as in [74].

To address the aggregate optimization, we initially investigated use of a modified version of the Markov chain model described in the first section; however, we found that it is unsuitable for describing the behavior of the aggregate system when it is repeatedly pushed to its constraints, as is done in arbitrage [71]. Therefore, we propose modeling the TCL population as a time-varying thermal battery, described more fully in [71]. This model keeps track of a TCL population’s energy state, \( S(k) \), as a function of its mean aggregate power usage, \( P_{\text{agg}} \), in each price interval, \([t_k, t_{k+1}]\), of width \( \Delta T \). The energy state of the population is defined similar to a battery’s state of charge; it describes how full an energy storage unit is. We can obtain a difference equation for the evolution of the energy state as well as upper and lower envelopes of achievable power and energy...
Figure 2.24: A TCL population’s baseline and power constraints. The TCL population also has energy constraints, not shown.

for a population of TCLs.

Without external control, a TCL population’s time-varying power trajectory is referred to as its “baseline.” Figure 2.24 shows an air conditioner population’s mean aggregate power baseline, $P_{agg, \text{baseline}}$, over a day. A TCL population increases its energy state when $P_{agg}(k) > P_{agg, \text{baseline}}(k)$, and decreases it when $P_{agg}(k) < P_{agg, \text{baseline}}(k)$:

$$S(k + 1) = S(k) + (P_{agg}(k) - P_{agg, \text{baseline}}(k)) \Delta T.$$  (2.43)

As shown in Fig. 2.24, the choice of $P_{agg}(k)$ is constrained:

$$P_{agg, \text{min}}(k) \leq P_{agg}(k) \leq P_{agg, \text{max}}(k).$$  (2.44)

The energy state is also constrained at each time step:

$$0 \leq S(k) \leq S_{\text{max}}(k).$$  (2.45)

These bounds define the power and energy capacity of a TCL population. When $S = 0$ the thermal battery is depleted meaning all TCLs operate at one edge of the dead-band (e.g., for cooling TCLs all operate near $\theta_{\text{set}} + \delta/2$). When $S = S_{\text{max}}$, the thermal battery is full meaning all TCLs operate at the other edge of the dead-band.

To use this model, we need to derive or identify the time-varying parameters: $P_{agg, \text{baseline}}$, $P_{agg, \text{min}}$, $P_{agg, \text{max}}$, and $S_{\text{max}}$. These parameters are a function of ambient temperature dynamics, but for simplicity we assume that each is simply a function of the current ambient temperature, $\theta_a$. Additionally, we assume that each belongs to a finite set of values and develop a look-up table that specifies an estimate of each value as a function of $\theta_a$. In recent work, we described the procedure for computation and estimation of
Given the thermal battery model and the price and outdoor air temperature forecasts over a horizon, we aim to determine the optimal mean aggregate power consumption in each interval, $\bar{P}_{agg}^*$, and so we solve:

$$\min \Delta T \sum_{k=t_0}^{t_0+N} l(k) \bar{P}_{agg}(k) \quad \text{(2.46)}$$

subject to (2.43), (2.44), and (2.45).

The above can be solved as a receding-horizon Linear Program (LP). We then transform $P_{agg}^*$ into a control trajectory $p_{agg}^*(t)$: $p_{agg}^*(t) = P_{agg}^*(k)$ for $t = k, k + h, ..., k + \Delta T - h$, to be tracked by the TCL population. Thus, we use the less accurate time-varying thermal battery model for the purpose of optimization over a long time horizon, while we use the Markov chain model for controlling TCLs to track the power output.

In order to track the trajectory $p_{agg}^*(t)$, we calculate $u_{goal}$, the total fraction of TCLs to switch on or off in the next time step. To achieve this, we use an extension of the Markov chain model of the heterogeneous population with a predictive proportional controller (PPC) [74]. The Markov chain model is extended in order to address large ambient...
temperature variations over long horizons. The extensions include addition of extra bins outside the temperature dead-band and identification of the $P$ matrix in (2.32) for every discretized ambient temperature. Here, the stochastic noise term $W$ is not explicitly considered in control synthesis.

To design the control, first, we compute:

$$u'_{\text{goal}}(t) = \frac{P_{\text{agg}}^*(t+1) - y(t+1)}{N_p P_{\text{rate}}},$$

where $y(t+1)$ is the predicted power output of the TCL population given the Markov chain abstraction as defined in Equation (2.32). Then, $u_{\text{goal}}(t)$ is calculated by putting $u'_{\text{goal}}(t)$ through a saturation filter with minimum equal to the fraction of TCLs on, and maximum equal to the fraction of TCLs off. We then distribute $u_{\text{goal}}$ to the bins and, for each bin, divide the absolute fraction of TCLs to switch by the measured or estimated fraction of TCLs in the bin to determine the switch probability. Switch probabilities are broadcast to the TCLs and then TCLs switch or not based on the switch probability associated with the bin they are in. Note that $u_{\text{goal}}$ can be distributed to the bins in different ways, for example, equally or by preferentially switching TCLs that are about to switch. Here we do the latter, so that the controller would preferentially switch TCLs in bins closer to the dead-band and thus to natural switching. This helps minimize the chance of compressor short-cycling.

### Results of Aggregate TCL Optimization

![Graphs showing comparisons of electricity prices, aggregate power, and optimal trajectory with and without control.]

**Figure 2.26:** Comparison of optimization by individual TCLs and with the thermal battery model.

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We use the same population of central air conditioners as in the individual TCL arbitrage population, and consider the same 10 hour period. However, here we use a stochastic individual TCL model (2.30) and assume the noise standard deviation is $5 \times 10^{-4}$. For implementation of control, we use 42 bins in the Markov chain: 40 bins within the dead-band and 2 bins to capture temperatures just outside the dead-band into which TCLs switch. Again, we assume perfect price and temperature forecasts. We identified $\bar{P}_{\text{agg, baseline}}, \bar{P}_{\text{agg, min}}, \bar{P}_{\text{agg, max}}, S_{\text{max}},$ and $P$ for integer values of temperature using the system identification methods described in [71].

First, in Fig. 2.26, we compare the individual TCL optimization results to the aggregate population optimization results. In the individual optimization, each TCL in the population optimizes its power consumption using the DP approach described above. In the aggregate population optimization, the optimal aggregate power trajectory is found using the thermal battery model and the LP optimization. Then, the TCL population tracks this desired power output using the on/off broadcast control described in the previous section. The aggregate results indicate savings of 21% in contrast to the 28% savings given by the individual optimization. In addition to apparent inaccuracy in the aggregate model compared to individual TCL model, a reason for the decreased savings is that the battery storage model based on the 5-minute discretization step results in a piecewise constant power trajectory, which may not allow the TCL population sufficient flexibility. In fact, tracking this trajectory is difficult for the TCL population, requiring significant TCL switching. Future work will explore other methods for transforming the output of the LP into more suitable control trajectories.

To address the potential for TCL populations to arbitrage energy prices using the aggregate population optimization framework, we performed one optimization each day, i.e. $N = 24$ hours for one full year (2010) of the same data source. Our results were discussed in detail in our recent publication [71], and here we summarize them. The results of the optimization problem predict that a population of air conditioners in Merced, CA, USA could save, at most, 17% in yearly wholesale energy cost through arbitrage in CAISO’s 5-minute energy market. When we control the population to track the optimal trajectory, we find that the maximum savings are closer to 14%, specifically the uncontrolled population would have spent about $91,500 for energy during the year while the same population doing energy arbitrage would have spent $78,400. This translates to about $13 in wholesale energy cost savings per TCL per year. Since this analysis assumes perfect price and weather forecasts and exogenous electricity prices, this is an upper bound on the potential practical energy costs savings in Merced, assuming future prices and price volatility are similar to those in the past.

2.2.6 Discussion of the Approaches

The formal Markov chain abstraction in the first section provides insight into the previously proposed models [13, 50, 53, 72, 74]. Its basis is dividing the continuous state into bins and associating a Markov chain state to each bin for each discrete on/off mode. The problem is then characterizing the evolution of the fraction of TCLs in each bin. The contribution here is to provide exact characterization of the evolution of the TCL
fractions in each bin for the homogenous system. The mean of the distribution is consistent with previously derived results [74]. The method also exactly characterizes the covariance of the distribution. In addition, it shows that as the number of TCLs grow, the evolution of the distribution approaches a linear system with additive Gaussian noise and the noise covariance is exactly characterized. This analysis leads to derivation of error bounds on the power consumption of the population and that predicted by the model. The error bound is a function of the number and length of the bins. Thus, it provides a method to tune the discretization in order to achieve a desired error behavior as well providing worst-case analysis of the error in the abstraction. The analysis is extended to a heterogeneous population of TCLs in which the model parameters for each TCL are drawn from a known distribution. The Markov chain abstraction can be used in a receding horizon approach to find the temperature set-point variation required for tracking a desired power. In addition, the linear Gaussian formulation of the dynamics leads itself naturally to Kalman filter estimation approach to estimate the state of the system given noisy measurements [74]. Given that the prediction error grows as the time horizon grows, the model although useful for optimal tracking and estimation over short time steps, has limitations for longer time steps.

The approach in Model 2 provides a fresh look at the problem of TCL population analysis. In particular, the approach is not based on developing a dynamical model of the TCL population. Rather, it uses observations of the TCL population power consumption to define bounds on the number of the TCLs within each bins, without solving for the exact values of these numbers. Then, the analysis problem of whether a given power trajectory can be tracked with a desired accuracy is cast as a feasibility problem, with the additional constraints given by the bounds on the number of TCLs in each bin defined by the historic data of population power. This particular line of analysis is important in applications in which the TCL population need to provide guarantees on the power tracking performance, such as ancillary services participation. Thus, the approach proposed could determine if there exists any feasible controller which satisfies ancillary service requirements. In theory, the model can address any form of uncertainty in the dynamics, such as uncertainty in the resistance values of the TCLs, through introducing non-determinism. The additional non-determinism maps to additional constraints in the feasibility problem under consideration. However, the limitation of the model is that as uncertainty dimension grows, the constraints lead to a very conservative approach which quickly leads to infeasibility of the problem even though in practice a feasible controller may exist. Thus, at this point, the model is not able to handle parameter heterogeneity and stochasticity.

Model 3 was developed with the aim of capturing heterogenous TCL behavior over long time horizons so that we could study the potential for TCLs to arbitrage energy prices in a realistic setting. The model needs to provide predictability of power consumption over horizons of minutes to hours in order to be able to take advantage of the temporal price differences which are of the same order of time magnitude. In order to deal with the above two issues, time-varying lower and upper bounds on achievable power and energy by a TCL population are derived. The power trajectory is designed to minimize energy costs over a prediction horizon while satisfying these bounds. The resulting optimal
trajectory can be tracked by the TCLs by broadcasting control signals to the population that cause TCLs to switch on/off probabilistically. The Markov chain abstraction is used to develop the controller. We find that this approach leads to about 75% of the savings achievable with the fully optimal approach of individual TCL arbitrage. The main limitation of this model is the fact that it is difficult to characterize the power and energy capacities. Here, we approximate them through system identification. More in depth analysis of these quantities are subject of current investigation.

2.3 N-1 Security and Reserve Scheduling

2.3.1 Introduction

The expected increase in the installed wind power capacity and other fluctuating power sources as well (e.g. photovoltaic power), highlights the necessity of revisiting certain operational concepts, like security and reserve scheduling. In a deterministic set-up, security of a power system refers to its ability to withstand contingencies without disruption of service [57], [77]. As a security measure, the N-1 security criterion is commonly used, under which the system is considered to be secure if any single component outage does not lead to any operational violations.

In the absence of uncertainty, many methods dealing with security enhancement have been proposed [95], [112], [47], [48], [106]. From a market point of view, the authors of [75] propose a method for incorporating contingencies and stability constraints by making use of a voltage constrained optimal power flow. On the other hand, in the presence of uncertainty most of the research has either concentrated on the economic implications of security [21], [44], or has resorted to Monte Carlo based statistical analyses [8], [83]. Toward maximizing the expected social welfare, optimization of reserve power has been addressed in [40], [23], [24], [22], in a security constrained market clearing context. Using the same framework, [76], [84] formulated a multi-stage stochastic unit commitment program, modeling the uncertain generation by means of scenarios and using reduction techniques to ensure tractability of the problem. However, these approaches do not offer a priori guarantees regarding the reliability of the resulting solution.

Following [103], we propose a unified framework that simultaneously solves the problem of designing an N-1 secure day-ahead dispatch for the generating units, while determining the minimum cost reserves and the optimal way to deploy them. To account for the variability of wind power we follow a probabilistic methodology, providing certain guarantees regarding the satisfaction of the system constraints. We first integrate, as in [102], the security constraints emanating from the N-1 criterion to a DC optimal power flow problem and formulate a stochastic optimization problem with chance constraints. Modeling the steady state behavior of the secondary frequency controller leads to a representation of the reserves as a linear function of the total generation-load mismatch, which may be due either to the difference between the actual wind and its forecast, or to a generator/load loss. We introduce different ways to distribute reserves based on the type of mismatch, thus offering an implementation of corrective security. The generation dispatch and the reserve capacity determination consist preventive control
actions, whereas the case dependent strategy according to which we deploy reserves in real time operation falls in the framework of corrective control. Apart from the physical intuition, using such a strategy for the reserves has the advantage that the number of decision variables does not grow with the number of uncertainty realizations as in [70] and the resulting solution is less conservative compared to [69]. This makes our method applicable even for large scale networks.

The resulting problem is a chance constrained, bilinear program. To achieve tractability, the issues arising due to the bilinear terms and the presence of the chance constraint need to be resolved. To alleviate these difficulties we propose a heuristic algorithm and a convex reformulation, and use the scenario based technique reported in D2.4 to deal with the chance constraint. To investigate the performance of the scenario based technique we use an approach based on the quantiles of the stationary distribution of the wind power error. The effectiveness of the proposed methodologies is illustrated by means of Monte Carlo simulations for a modified version of the IEEE 30-bus network [113].

The rest of the chapter is organized as follows. In Section 3.2 we formulate the security constrained reserve scheduling problem as a chance constrained optimization program. Section 3.3 provides details on how to deal with the bilinearity problem and the chance constraint and Section 3.4 illustrates the performance of the proposed approaches via a simulation study. Finally, in Section 3.5 we provide some directions for future work.

2.3.2 Problem formulation

Problem set-up and definitions

We consider a power network comprising $N_G$ generating units, $N_L$ loads, $N_l$ lines, and $N_b$ buses. For the security constrained formulation, as in [40], [38], we take into account any outage involving the tripping of a branch, generator or load, and by $N_{out} = N_l + N_G + N_L$ we denote the total number of possible single outages. Moreover, denote by $I_l, I_L, I_G$ the set of indices corresponding to branch, load and generator outages. The “0” index corresponds to the case of no outage. Let also $I = \{0\} \cup I_l \cup I_L \cup I_G$.

The problem under study is based on the following assumptions: 1) A standard DC power flow approximation [10] is used. 2) Wind generation is located at a single bus of the network. 3) Perfect load forecasts are considered. 4) Line outages do not lead to multiple generator/load failures. 5) The “on-off” status of the generating units has been fixed a-priori by solving a unit-commitment problem. The first assumption is standard for this type of problems. The second, third and fourth are included to simplify the presentation of our results and could still be captured by the proposed algorithm. Removing the last assumption by incorporating the unit commitment problem would give rise to a mixed-integer problem; this can be tackled using the modified version of the scenario approach [68] as discussed in [70].

Under the DC power flow approximation and by eliminating the angles by setting the reference angle to zero [102], the vector including the power flows across each line can be defined as $P_{fi} = B_{fi}^T \left( B_{BUS}^T \right)^{-1} \tilde{P}^i \in \mathbb{R}^{N_l}$, where the power injection vector $\tilde{P}^i$
is given by
\[
\tilde{P}^i = \left[ I_i^G C_G (P_G + R_i) + C_w P_w - I_i^L C_L P_L \right]_{N_b-1} \in \mathbb{R}^{N_b-1},
\]
where \([\cdot]_{N_b-1}\) denotes the first \(N_b - 1\) rows of the quantity inside the brackets. For every outage, matrices \(B_{fi}^i\) and \(B_{BUS}^i\) denote the imaginary part of the admittance of each network branch and the nodal admittance matrix. \(P_G \in \mathbb{R}^{N_G}\), \(P_w \in \mathbb{R}\), and \(P_L \in \mathbb{R}^{N_L}\) denote the generation dispatch, the wind power in-feed and the load, respectively. \(R_i \in \mathbb{R}^{N_G}\) is a power correction term, which is related to the reserves provided by each generator and will be defined in the sequel. Matrices \(C_G, C_w, C_L\) are of appropriate dimension, and their element \((i,j)\) is “1” if generator \(j\) (respectively wind power/load) is connected to the bus \(i\) and zero otherwise. Matrices \(I_i^G \in \mathbb{R}^{N_b \times N_b}\), \(I_i^L \in \mathbb{R}^{N_b \times N_b}\) depend on the outage \(i\). Specifically, for \(i \in \{0\} \cup \mathcal{I}_l \cup \mathcal{I}_L\) (i.e. the case where we have no generator outage), \(I_i^G \in \mathbb{R}^{N_b \times N_b}\) (similarly for \(I_i^L \in \mathbb{R}^{N_b \times N_b}\)) is an identity matrix, where for \(i \in \mathcal{I}_G\) the diagonal element with index corresponding to the bus to which the tripped generator is connected is set to zero.

**Reserves representation**

Reserves are needed to balance generation-load mismatches, which may occur due to a difference between the actual wind power and its forecast, or as an effect of a generator/load loss. Such mismatches between load and generation induce frequency deviations and activate the primary and secondary frequency controller (by means of Automatic Generation Control - AGC). Here we assume an ideal primary frequency control functionality compensating for any fast time scale power deviation and focus only on the steady state behavior of the AGC actions (hence on the secondary frequency control reserves). The AGC output is distributed to certain participating generators, whose setpoint is changed by a certain percentage of the active power to be compensated. The product of these percentage weights with the worst case imbalance results in the amount of reserves that each generating unit has to provide. We will refer to the vector that includes these weights as the *distribution vector*.

The existing set-up of the AGC loop is shown in Fig. ??, demonstrating the role of the distribution vector. In current practice this vector results from the market that determines the secondary frequency control reserves and remains constant until the next market auction. Typically, this task is performed without taking the network constraints into account. Moreover, the distribution vector may differ between up-spinning and down-spinning reserves, but is the same for all possible outages. In this paper, in view of a corrective security control scheme, apart from distinguishing between up-spinning and down-spinning reserves, we also consider different distribution vectors depending on the outage. Optimizing then over the distribution vectors we determine an optimal reserve schedule, while taking the network security constraints into account. Our approach enables us to compute simultaneously both the minimum cost reserves per generator, and also a reserve strategy that can be deployed in real time operation. This strategy consists in using the distribution vectors, which depending on the outage and the wind
power deviation dictate the amount of power by which each generating unit should adjust its production. Therefore, the proposed methodology serves as an alternative to other methods for reserve scheduling, e.g. [76], [84], which account implicitly for real time response via their day-ahead decisions.

To encode the proposed reserve representation, we define a power correction term $R_i$ as a linear function of the total generation-load mismatch. This term is directly related to the reserves since it shows the amount of the reserves that, for every mismatch, will be provided by each generator.

$$R_i = d_{i,up}^{\text{max}} - P_m^i + d_{i,down}^{\text{max}} - P_m^i, \text{ for all } i \in \mathcal{I},$$

where $\max(\cdot) = \max(\cdot, 0)$. Variable $P_m^i \in \mathbb{R}$ denotes the generation-load mismatch, which for each outage is given by

$$P^i_m = \begin{cases} P_w - P_w^f & \text{if } i \in \mathcal{I}_l \text{ or } i = 0 \\ P_w - P_w^f + P_L^i & \text{if } i \in \mathcal{I}_L \\ P_w - P_w^f - P_G^i & \text{if } i \in \mathcal{I}_G \end{cases}$$

Note that $P_L^i, P_G^i \in \mathbb{R}$ denote the element of $P_L \in \mathbb{R}^{N_L}$, $P_G \in \mathbb{R}^{N_G}$, that corresponds to the failed component $i$. Vectors $d_{up}^i, d_{down}^i \in \mathbb{R}^{N_G}$ represent the distribution vectors. The sum of their elements is equal to one and, if a generator is not contributing to the AGC, the corresponding element in the vector is zero. The indices up and down are used to distinguish between up-spinning reserves and down-spinning reserves.

If $P_m^i$ is negative, up-spinning reserves are provided and the production of the generators is increased accordingly, whereas in the opposite case the second term of (2.49) is active and hence down-spinning reserves are provided. Notice that $d_{up}^i, d_{down}^i, i \in \mathcal{I}$ may have negative elements as well. Consider for example the base case where we have no outages, the power mismatch is negative ($P_m^{0} < 0$) and some elements of $d_{up}^0$ are negative as well. This corresponds to a set-up where the network is congested. The interpretation of some elements of $d_{up}^i$ being negative is that the corresponding generators should provide down-spinning reserves so that congestion is relieved, while the rest of the units would provide up-spinning reserves.

**Probabilistic security constrained reserve scheduling**

We consider an optimization horizon $T = 24$ with hourly steps and introduce the subscript $t$ to indicate the value of the quantities for a given time instance $t = 1, \ldots, T$. We consider a quadratic form for the production cost and a linear cost for the reserves. Let $C_1, C_2, C_{up}, C_{down} \in \mathbb{R}^{N_G}$ be generation and reserve cost vectors and let $[C_2]$ denote a diagonal matrix with vector $C_2$ on the diagonal.

For each step $t$ define $x_t = [P_G^t, d_{up}^t, d_{down}^t, R_{up}^t, R_{down}^t]_{i \in \mathcal{I}}^T \in \mathbb{R}^{N_G + 4N_G(N_{out} + 1)}$ to be a vector of decision variables, where $R_{up}^t, R_{down}^t \in \mathbb{R}^{N_G}$ denote the probabilistically worst-case up-down spinning reserves that the system operator needs to purchase.
for every $i \in \mathcal{I}$. The resulting optimization problem is

$$
\min_{\{x_i\}_{i=1}^T} \sum_{t=1}^T \left( C^T_{G,t} P_{G,t} + P^T_{G,t} [C_2] P_{G,t} + \sum_{i=0}^{N_{s acquisitions}} (C^T_{up} R^i_{up,t} + C^T_{down} R^i_{down,t}) \right),
$$

subject to

1) Forecast power balance constraints: For all $t = 1, \ldots, T$,

$$
1^T (C^G P_{G,t} + C^w P^f_{w,t} - C^L P_{L,t}) = 0.
$$

This constraint encodes the fact that the power balance in the network should always be satisfied when $P_{w,t} = P^f_{w,t}$.

2) Generation limits: For all $t = 1, \ldots, T$,

$$
P_{\text{min}} \leq P_{G,t} \leq P_{\text{max}},
$$

where $P_{\text{min}}, P_{\text{max}} \in \mathbb{R}^{N_G}$ denote the minimum and maximum generating capacity of each unit.

3) Distribution vector constraints: For all $t = 1, \ldots, T$ and for all $i \in \mathcal{I}$

$$
1^T d_{up,t}^i = 1, 1^T d_{down,t}^i = 1,
$$

For $i \in \mathcal{I}_G$, the element of $d_{up,t}^i, d_{down,t}^i$ corresponding to the tripped generator is equal to zero. Constraints (2.54) encode the fact that the elements of the distribution vectors should sum to one.

4) Probabilistic constraints: For all $t = 1, \ldots, T$,

$$
P \left( P_{w,t} \in \mathbb{R} \mid -\mathcal{P}^i_{\text{line}} \leq B^i_{ft} \begin{bmatrix} (\mathcal{B}^i_{\text{BUS}})^{-1} \mathcal{P}^i_{t} \\ 0 \end{bmatrix} \leq \mathcal{P}^i_{\text{line}} \right) \leq 1 - \epsilon,
$$

where the probability is meant with respect to the probability distribution of the wind power vector $P_{w,t} \in \mathbb{R}$. The first constraint inside the probability denotes the standard transmission capacity constraints for each outage $i$. $\mathcal{P}^i_{\text{line}}$ represents either normal or emergency line ratings. The second constraint provides guarantees that the scheduled generation dispatch plus the power correction term $R^i_t$ will not result in a new operating point outside the generation capacity limits. The last constraint in (2.55) is included to determine the reserves $R^t_{up,t}, R^t_{down,t}$ as the worst case, in a probabilistic sense, value of the power correction term $R^i_t$. The reserves that the system operator will need to purchase are then determined as $R_{up,t} = \max_{i \in \mathcal{I}} R^i_{up,t}$ and $R_{down,t} = \max_{i \in \mathcal{I}} R^i_{down,t}$, which denote the worst case values of $R^i_{up,t}$ and $R^i_{down,t}$, respectively. Note that in (2.55) we considered the same probability level $\epsilon$ for each time-step $t = 1, \ldots, T$, but different
probability levels per stage or a joint chance constraint for all stages could be captured by the proposed framework as well.

Following this formulation we propose an additional AGC functionality. The operator of the system needs to monitor both the production of the tripped plant and the deviation of the wind power from its forecast, and using (2.49) as a look-up table, select the appropriate distribution vector, among those computed in the optimization problem (see Fig. ??).

The resulting problem (2.48)-(2.55) is a chance constrained bilinear program whose stages are only coupled due to the temporal correlation of the wind power. We could have a further coupling among the stages if a unit commitment problem was included or if ramping constraints of the generating units and minimum up and down times were modeled. The reader is referred to [70] for a set-up where all of these constraints are included. There are two main challenges when attempting to solve problem (2.48)-(2.55). The first arises from the presence of bilinear terms due to the products of $d_{i,\text{up},t}$, $d_{i,\text{down},t}$ and $P_{G,t}$ for $i \in I_G$, whereas the second is due to the presence of the chance constraint.

### 2.3.3 Tractable problem reformulations

#### Method 1: Heuristic algorithm

We propose here a method based on an iterative algorithm (Algorithm 2) to deal with the bilinear terms. We first attempt to identify a feasible solution to the problem starting from an arbitrarily chosen power schedule $P_{G,t}^0$. At iteration $k$ of the algorithm, we fix $P_{G,t}^k$ only in (2.49) to the value obtained in the previous iteration. Therefore, $R^k$ is still a function of the distribution vectors and the production, but this time the value of the power production term is fixed to $P_{G,t}^0$ to avoid the presence of bilinear terms. Solving then (2.48)-(2.55) a new solution $x_t^k$ is computed and $P_{G,t}^k$ is updated accordingly. If the algorithm converges, its fixed point $x_t^k$ will be a feasible solution.

At a second step, we use an alternating scheme to refine the resulting feasible solution in terms of cost. At iteration $k$ we first fix $d_{i,\text{up},t}^k, d_{i,\text{down},t}^k$ to the values obtained at the previous step of the algorithm only for $i \in I_G$ and by solving (2.48)-(2.55) obtain

$$[P_{G,t}^k, [d_{i,\text{up},t}^k, d_{i,\text{down},t}^k]_{i \in I_G}, [R_{i,\text{up},t}^k, R_{i,\text{down},t}^k]_{i \in I_G}]^T.$$  

We then fix $P_{G,t}^k$ to the computed value in all equations it appears and solve for $[d_{i,\text{up},t}^k, d_{i,\text{down},t}^k, R_{i,\text{up},t}^k, R_{i,\text{down},t}^k]_{i \in I_G}$. The entire process is then repeated until convergence. Note that the first part of Algorithm 1 is a heuristic scheme applied to identify a feasible solution and no convergence guarantees can be provided. The second part of the algorithm converges monotonically (this is not necessarily the case for the first part), since it is a bilinear descent iteration; however, the limit point is not guaranteed to be the global optimum of the original bilinear problem.

Fig. 2.27 shows how the power dispatch of each unit and the obtained objective value change per iteration for the benchmark problem introduced in the next section. After 3 iterations the first part converges, whereas for the second only one iteration is needed. As expected, the cost decreases monotonically in the second part.
Figure 2.27: Illustration of Algorithm 2 for one hour of the simulated data, initialized with $P_{G,t}^0 = 0$. For the first part, the power dispatch of each unit and the obtained objective value converge after 3 iterations, whereas for the second only 1 iteration is needed.

Method 2: Convex reformulation

Assume that in the case where $i \in \mathcal{I}_G$ we can distinguish between the mismatch that corresponds to wind deviation and the one that occurs due to a generator outage by introducing different distribution vectors. For $i \in \mathcal{I}$, the power correction term would now be $R_i^t = d_{up,t}^2 \max_+ (P_{w,t}^f - P_{w,t}) - d_{down,t}^1 \max_+ (P_{w,t}^f - P_{w,t}) + d_{up,t}^1 P_{G,t}^i$. By considering the optimization problem that corresponds to (2.48)-(2.55) if the additional distribution vectors are introduced, $d_{up,t}^2 P_{G,t}^i$ becomes the only bilinear term, which appears both in the constraints and the objective function. Setting $z_i^t = d_{up,t}^2 P_{G,t}^i \in \mathbb{R}^{N_G}$ and defining the new decision vector $\tilde{x}_t = [P_{G,t}, [d_{up,t}^1, d_{down,t}^1]_{i \in \mathcal{I}_G}, [d_{up,t}^1, d_{down,t}^1]_{i \in \mathcal{I}_G}, [R^i_{up,t}, R^i_{down,t}]_{i \in \mathcal{I}_G}]^T \in \mathbb{R}^{N_G + N_G^2 + 4 N_G (N_{out} + 1)}$, the resulting problem is linear in $z_i^t$ and hence convex. It is of the same structure as (2.48)-(2.55) with the additional constraint

$$1^T z_i^t = P_{G,t}^i, \text{ for all } i \in \mathcal{I}_G. \quad (2.56)$$

Once the solution to this problem is computed, for all $i \in \mathcal{I}_G$, $d_{up,t}^2 P_{G,t}^i$ is calculated as $d_{up,t}^2 = z_i^t / P_{G,t}^i$ if $P_{G,t}^i$ is not equal to zero and is set to zero otherwise. Note that the sum of the elements of $d_{up,t}^2$ is constrained to be one, since $z_i^t, i \in \mathcal{I}_G$ satisfies (2.56). Given a mismatch $P_{m,t}^i = (P_{w,t} - P_{w,t}^f) - P_{G,t}^i$, the participation of each unit in compensating $P_{m,t}^i$ can be computed as $R^i_{up,t} / 1^T R^i_{up,t}$. Note that following this procedure we convexify the bilinear terms inside the probability. The overall problem is still non-convex, however,
Algorithm 2.3.1
1: Initialization – Part 1.
2: Set $P_{G,t}^0$ (e.g. $P_{G,t}^0 = 0$),
3: $k = 1$.
4: Repeat until convergence
5: Set $P_{G,t}^k = P_{G,t}^{k-1}, \forall i \in \mathcal{I}_G$, only in (2.49),
6: Compute $x_t^k$ solving (2.48)-(2.55),
7: Update $P_{G,t}^k$,
8: $k = k + 1$.
9: end
10: Return converged solution $x_t^{k^*}$
11: Initialization – Part 2.
12: Set $d_{\text{up},t}^{0,i} = d_{\text{up},t}^{k^*,i}$, $d_{\text{down},t}^{0,i} = d_{\text{down},t}^{k^*,i}$, $\forall i \in \mathcal{I}_G$,
13: $k = 1$.
14: Repeat until convergence
15: Set $d_{\text{up},t}^{k,i} = d_{\text{up},t}^{k-1,i}$, $d_{\text{down},t}^{k,i} = d_{\text{down},t}^{k-1,i}$, $\forall i \in \mathcal{I}_G$, in (2.49),
16: Compute $[P_{G,t}^k, [d_{\text{up},t}^k, d_{\text{down},t}^k]_{i \in \mathcal{I}_G}, [R_{\text{up},t}^k, R_{\text{down},t}^k]_{i \in \mathcal{I}}^T]$ solving (2.48)-(2.55),
17: Fix $P_{G,t}^k$ in (2.48)-(2.55),
18: Compute $[d_{\text{up},t}^{k,i}, d_{\text{down},t}^{k,i}, R_{\text{up},t}^{k,i}, R_{\text{down},t}^{k,i}]_{i \in \mathcal{I}}^T$ solving (2.48)-(2.55),
19: $k = k + 1$.
20: end

due to the presence of the chance constraint. In Section III-C we elaborate on how to deal with this issue.

Solving the chance constrained problem - The scenario approach

Applying method 2 directly transforms (2.48)-(2.55) to a problem where the constraints inside the chance constraints are convex. This is also the case at every iteration of method 1, with the difference that the size of the resulting problem might be different from that of method 2. In this section we provide a scenario based methodology to deal with the chance constraint while ensuring tractability of the resulting program. Since the decision variables of the different optimization stages are not coupled via the constraints or the objective function, we can treat each hour separately. The chance constrained problem that we need to solve can be then written in a more compact notation as

$$\min_{x \in \mathbb{R}^{N_x}} J(x)$$

s.t.: $\mathbb{P}\left(\delta \in \Delta \mid A(\delta)x + c(\delta) \leq 0\right) \geq 1 - \epsilon,$

$$\text{(P1)}$$

where $x \in \mathbb{R}^{N_x}$ is the vector of decision variables with $N_x$ being the dimension of $x_t$, $\delta \in \Delta \subset \mathbb{R}$ is the uncertain parameter (in this case the wind power $P_{w,t}$), $J(x) \in \mathbb{R}$ is quadratic in $x$, and $A, c$ are of appropriate dimension. Using the standard scenario
approach of [27] requires replacing the chance constraint with a finite number of hard
constraints, each of them corresponding to a different uncertainty realization. This
results in the following optimization problem:

\[
\begin{align*}
\min_{x \in \mathbb{R}^{N_x}} & \quad J(x) \\
\text{s.t.:} & \quad A(\delta^{(k)})x + c(\delta^{(k)}) \leq 0 \text{ for } k = 1, \ldots, N,
\end{align*}
\]

where \(\delta^{(k)}, k = 1, \ldots, N\) denote the different realizations of the uncertain parameter
\(\delta\), extracted according to \(\mathbb{P}\). The authors of [32] provide a bound on number \(N\), of
uncertainty realizations that one needs to generate to achieve probabilistic constraint
violation guarantees. It is shown that if we generate

\[
N \geq \frac{2}{\epsilon} \left( \ln \frac{1}{\beta} + N_x \right),
\]

realizations of the uncertainty, the resulting solution of \((P'_1)\) will satisfy the chance
constraint in \((P_1)\) with probability at least \(1 - \beta\); here \(N_x\) is the number of decision
variables and \(\beta \in (0, 1)\) is a confidence parameter. There are two basic limitations of this
procedure. The number of scenarios (and hence also the number of constraints in \((P'_1)\))
grows with respect to the number of decision variables and convexity of the objective
function and the constraints of the initial problem is required. The latter prevents
us from providing probabilistic performance guarantees to mixed-integer programs like
those arising in unit commitment problems.

To overcome this difficulty we exploit the recent results of [68]. Instead of directly
using the scenario approach and solving \((P'_1)\), we construct and solve a robust version
of \((P_1)\) with interval bounded uncertainty, where the uncertainty bounds are computed
at an intermediate step using the scenario approach. Specifically, we seek bounds \(\tau = (\underline{\tau}, \overline{\tau}) \in \mathbb{R}^2\), such that \(\delta \in [\tau, \tau]\) with probability at least \(1 - \epsilon\). Therefore, consider the
optimization problem

\[
\begin{align*}
\min_{\tau \in \mathbb{R}^2} & \quad (\tau - \underline{\tau}) \\
\text{s.t.:} & \quad \mathbb{P} \left( \delta \in \Delta \mid \delta \in [\tau, \tau] \right) \geq 1 - \epsilon.
\end{align*}
\]

This is a different chance constrained problem, whose objective function and constraints
are convex by construction. Therefore, we can determine its solution using the scenario
approach. This requires solving the following problem:

\[
\begin{align*}
\min_{\tau \in \mathbb{R}^2} & \quad (\tau - \underline{\tau}) \\
\text{s.t.:} & \quad \delta^{(k)} \in [\tau, \tau] \text{ for } k = 1, \ldots, \tilde{N},
\end{align*}
\]

where \(\tilde{N} \geq \frac{2}{\epsilon} \left( \ln \frac{1}{\beta} + 2 \right)\) since \((P_2)\) has only two decision variables. Note that \((P'_2)\)
is effectively a selection problem and \(\tau, \underline{\tau}\) correspond to the maximum and minimum,
respectively, among the \(\tilde{N}\) generated samples. We then use the computed bounds to
formulate the robust counterpart of \((P_1)\),

\[
\begin{align*}
\min_{x \in \mathbb{R}^N_x} & \quad J(x) \\
\text{s.t.:} & \quad \max_{\delta \in [\tau, \overline{\tau}]} A(\delta)x + c(\delta) \leq 0.
\end{align*}
\tag{P_3}
\]

The maximization in \((P_3)\) is interpreted element-wise. Note that \((P_3)\) is no longer a stochastic program, but we require that its constraints be satisfied for all values of the uncertainty inside \([\tau, \overline{\tau}]\), where \(\tau, \overline{\tau}\) are computed by \((P'_2)\). Therefore, \((P_3)\) is a robust optimization problem (equivalently it could be thought of as a min-max problem) and can be solved efficiently using the algorithms of \[16\]. In particular, for the specific set-up where the uncertainty is scalar, it suffices to enforce the constraints in \((2.55)\) only for the extreme values \(\tau, \overline{\tau}\) of the uncertainty intervals, which correspond to the maximum and minimum, in a probabilistic sense, value of the wind power (see also \[70\]). Following \[68\], with confidence at least \(1 - \beta\), any feasible solution of \((P_3)\) satisfies the chance constraint of \((P_1)\). Since the chance constraint corresponding to each hour \(t = 1, \ldots, T\) is satisfied with confidence at least \(1 - \beta\), all chance constraints would be simultaneously satisfied with confidence at least \(1 - T\beta\) (see also \[67\]).

In contrast to the standard scenario approach, our methodology provides finite sample size guarantees regarding the probability of constraint satisfaction, convexity of the initial problem with respect to the decision variables. This implies that our method is applicable to all problems where \((P_3)\) can be solved effectively. This includes convex problems, but also problems such as classes of mixed integer programs, for which effective robust optimisation algorithms already exist \[16\]. This feature is exploited in \[70\], where the unit commitment problem is incorporated in the proposed framework.

Including additional uncertainty sources (e.g. more wind power generators, load uncertainty) or introducing coupling constraints among the stages of the optimization problem (ramping constraints, minimum up-down times, etc.) would result in a problem where the uncertainty is no longer scalar. The preceding formulation would remain unaffected with the exception that the number of scenarios \(N\) would depend on the dimension of the uncertainty vector and that problem \((P_3)\) would have hyper-rectangular instead of interval bounded uncertainty. Using the methodology of \[16\] the resulting optimization still leads to a tractable reformulation.

**Solving the chance constrained problem - The quantile based approach**

An alternative way to treat the chance constraint is to consider the stationary distribution of the wind power error. Two extreme scenarios are then considered: a low one corresponding to the forecast plus the \(\epsilon/2\) percentile of the error distribution and a high

\[12\] Including the unit commitment problem in the proposed probabilistic framework would require the introduction of a vector \(u \in \{0, 1\}^{N_G}\) of binary variables to encode the “on-off” status of the generating units. This would give rise to a term \(Bu\) additive to the left-hand side of the inequality inside the chance constraint of problems \((P_1)\), \((P'_1)\) and \((P_3)\), where \(B \in \mathbb{R}^{Nx\times N_G}\) would be a matrix with constant entries. The resulting problem \((P_3)\) would be a robust mixed-integer quadratic program since it would involve optimizing over both \(x\) and \(u\) and can be tackled using the algorithms of \[16\].
one corresponding to the forecast plus the \((1 - \epsilon/2)\) percentile of the error distribution (that way we have the same \(\epsilon\)-guarantees with the scenario approach). Now, treating the wind power as a bounded uncertainty, with the bounds corresponding to these two extreme cases, we compute the generation dispatch and the reserves by solving the robust counterpart of \((2.48)-(2.55)\), where the bilinear constraints are tackled using either method 1 or method 2. As in the previous approach, due to the particular structure of the problem it suffices to enforce the constraints in \((2.55)\) only for the two extreme values of the wind power, computed as described above. Note that the resulting problem is of the same type as the one that we need to solve when using the scenario approach of the previous section, with the difference that the extreme values of the wind power are not necessarily the same for both methods.

**Probabilistic performance guarantees**

Based on the method we use to deal with the bilinearity problem and the approach we adopt to tackle the chance constraint, we can offer different probabilistic performance guarantees. If method 1 is selected to deal with the bilinear constraints, we need to employ the scenario or the quantile based approach at every iteration of the heuristic algorithm. The resulting solution (if convergence at the first part of Algorithm 1 occurs) will be feasible by construction for the bilinear problem. Therefore, applying either the scenario or the quantile based approach we have probabilistic guarantees, that the resulting optimal solution is feasible for the initial chance constrained problem \((2.48)-(2.55)\).

Consider now the case where method 2 is used to transform the bilinear constraints to convex ones. Following the scenario approach results in solving the robust problem...
The solution of \((P_3)\) is feasible for \((P_1)\) with confidence at least \(1 - \beta\). However, we have no guarantees that the resulting solution is feasible for the bilinear chance constrained problem \((P_{2.48})-(P_{2.55})\). In the particular case where the uncertainty at every stage of the optimization problem is scalar, it is shown in [103] that using method 2 we obtain probabilistic guarantees regarding the satisfaction of the constraints of \((P_{2.48})-(P_{2.55})\).

### 2.3.4 Simulation study

#### Wind power model and simulation set-up

We assume that the wind power is the sum of a deterministic component (forecast) and a stochastic one, which models the error between the forecast and the actual wind power. To generate scenarios for the wind power error, we employed the approach of [82], which proposes a Markov chain model to generate wind power time series that take into account the temporal correlation of the wind power error. We used five-year, hourly measured data (both actual and forecasted values), for the aggregated wind power production of Germany over the period 2006-2011. Discretizing the wind power error with 41 states, we construct the transition probability matrix \(\bar{P}\) for the wind power error. It exhibits a pronounced block-triangular structure suggesting strong auto-correlation of the wind power error. The stationary distribution \(\pi\) of the wind power error is computed as a vector whose entries are all non-negative, sum up to one, and satisfy \(\pi = \bar{P}^T \pi\).

To evaluate the performance of our approach we applied it to the IEEE 30-bus network [113], which includes \(N_b = 30\) buses, \(N_G = 6\) generators, \(N_l = 41\) lines, and is modified to include a wind power generator connected to bus 22. All numerical values for the network data are retrieved from [113].

For all simulations we used \(\epsilon = 10^{-1}\) and \(\beta = 10^{-4}\). Note that when using the scenario approach, decreasing the value of \(\epsilon\) leads to a higher number of scenarios. However, using the scenario based methodology we only need the scenarios to solve Problem \((P_{2.48}^{\prime})\), which is effectively a selection problem and thus easy to solve. The size of problem \((P_3)\) is of importance and this is independent from the number of scenarios. The number of scenarios will just determine (in a probabilistic sense) the size of the interval uncertainty bounds and hence the conservatism of the resulting solution (the lower the value of \(\epsilon\) the more conservative the solution of \((P_3)\) tends to be). Therefore, our choice for \(\epsilon\) serves as a compromise between the theoretical guarantees we can offer and the conservatism of the resulting solution, as this is quantified by an a posteriori analysis and is not related to the computational cost of solving \((P_3)\).

To collect statistical results regarding the performance of our algorithm, we carried out a Monte Carlo study, evaluating the solution of \((P_{2.48})-(P_{2.55})\) (reformulated based on the proposed alternatives, i.e. method 1 and 2, scenario and quantile approach) against 10,000 wind power realizations, not included in the optimization process. Using the obtained reserve strategy (i.e. the power correction term \((P_{2.49})\) with the distribution vectors fixed according to the outcome of the optimization problem and the wind power equal to the evaluation scenario), for each of these realizations we examined whether the problem constraints are satisfied. Note that since we examine the feasibility of all
constraints, all possible outages are taken into account in the evaluation phase. Since we perform a probabilistic design, applying our reserve strategy still allows for constraint violation but with a certain probability. By constraint violation we mean the case where the wind power realization used to evaluate our solution leads to a power mismatch for which at least one of the constraints is violated. Such a violation does not necessarily correspond to the base case but to some $i = 0, \ldots, N_{out}$. In case we violate the constraints and end up with an excess of power, we refer to the maximal such amount as power surplus, which corresponds to a potential wind power curtailment action. In the opposite case we use the term power deficit to characterize the amount of power that may not be covered by the scheduled up-spinning reserves. In the realistic set-up of an interconnected system a fraction of this amount would be provided by the primary frequency reserves of neighboring areas (assuming the primary reserves of our area are also at saturation). If these primary reserves are not sufficient to cover the power deficit load shedding will occur. Following its definition, if no power deficit is encountered the system will always be N-1 secure.

Note that for the simulation study of the next section we differentiate among the distribution vectors based on the sign of the wind power error and the possible generator outage, thus having the same vector for all line and load outages. Our choice is motivated by a desire to minimize the number of decision variables (and hence the computational cost) in the optimization problem.

All optimization problems were solved using the solver CPLEX [50] via the MATLAB interface YALMIP [58].

Simulation results

We first investigate the performance of methods 1 and 2 when applying both the scenario approach and the quantile based approach for one day of our data. Fig. 2.29(a) shows the forecast (“blue”) and the actual (“red”) wind power, the wind power scenarios (“green”) that were used for the scenario approach (generated according to $\tilde{N}$), and the wind power quantiles that were used for the quantile based approach. For the analysis of Fig. 2.29(c) the solution of (2.48)-(2.55) was evaluated using the actual wind power realization (red curve in Fig. 2.29(a)). Fig. 2.30, 2.31 provide statistical information regarding the performance of our methods using 10,000 wind power realizations, that were used for evaluation purposes and their span corresponds to the shaded region of Fig. 2.29(a).

Fig. 2.29(b) shows the scheduled cost (production + reserves) for the convex reformulation (“dashed” line) and the heuristic algorithm (“solid” line), using both the scenario and the quantile based approach. All methods lead to a similar cost, but the quantile based ones result in slightly lower values. The difference in the cost occurs due to the fact that the quantile based approach leads to scheduling less reserves, since it does not capture extreme scenarios like the scenario approach. However, this is at the expense of more frequent constraint violations (see also Fig. 2.30, 2.31). The latter is highlighted in Fig. 2.29(c) which shows the amount of power deficit for each method, as this occurs once the actual wind is realized. Using the scenario approach neither method 1 nor method 2 lead to a power deficit, whereas the for the quantile based approach the
amount of deficit is the same for both methods. As discussed below, this is not the case in general. No power surplus was encountered since the actual wind scenario never exceeds the upper quantile barrier.

For comparison purposes we solved the nonlinear problem \((2.48)-(2.55)\) directly using the nonlinear solver IPOPT. In all cases the resulting solution led in slightly lower cost values compared to method 1, with a maximum difference of 1%. Therefore, method 1 provides a reliable alternative to more direct schemes based on nonlinear solvers, since it leads to a similar cost while involving the solution of a sequence of convex problems.

Method 2 leads to slightly lower cost compared to the nonlinear solver (maximum difference of at most 1%). This is due to the fact that for our set-up Method 2 provides an exact convex reformulation of the bilinear problem \([103]\). This implies that these problems have the same optimal objective values, however, we do not have guarantees that the bilinear one (solved using the nonlinear solver) can be solved up to optimality. In the general case, where the requirements of Proposition 1 are not satisfied, Method 2 will not necessarily outperform the solution of the nonlinear solver in terms of cost since it will only be a convex relaxation of the bilinear problem. However, since Method 2 leads to a convex problem the computational cost will be much lower.

Method 1 and 2 lead to different distribution of the reserves among the various generators and to different total amount of reserves in general. Therefore, the amount of power deficit or surplus differs according to whether method 1 or 2 is employed. Figs. 2.30(a),(c), 2.31(a),(c) show the power surplus and power deficit for the scenario approach and the quantile based approach respectively, when method 2 is used\(^{13}\). The results for method 1 are similar and are omitted in the interest of space. However, the probability of constraint violation (power deficit or surplus) depends solely on the wind power used for evaluation purposes. To see this, notice that the left or right hand-side inequalities of the last constraint inside \((2.55)\) will always hold with equality for at least one of its rows (possibly different rows for methods 1 and 2). From the definition of the power correction term, this implies that any wind power realization outside the span of the scenarios used for the scenario approach or the “band” of the quantile based approach will result in violating these specific constraints. Therefore, the distributions shown in Figs. 2.30(b),(d), 2.31(b),(d) are the same both for method 1 and 2. The probabilities of these figures are calculated as the fraction of the 10,000 evaluation scenarios that resulted in power surplus and deficit, respectively. The empirical probability of constraint violation is determined by the sum of the individual probabilities of power surplus and deficit. The quantile based approach, even though it provides the same \(\epsilon\)-type theoretical guarantees, leads to systematically more power deficit and surplus since the scenarios (“green”) span the entire range of the shaded region, which includes the wind power realizations that were used for evaluation purposes. It should be noted that load shedding and wind power curtailment becomes more prominent at later times due to accumulated forecast inaccuracy. This is more pronounced when using the quantile based approach

\(^{13}\)In the boxplots, the “red” line corresponds to the median value, the edges of the box correspond to the 25\(^{th}\) and 75\(^{th}\) percentiles, whereas the whiskers extend to a 99\% coverage. The “red” marks denote the data outliers, which lie outside the 99\% confidence region.
Table 2.7: Comparison of the proposed alternative approaches in terms of cost, power deficit, power surplus and the degree of conservatism.

(Fig. 2.31) since the robustness guarantees offered by this approach are limited in a band (“dashed” black lines in Fig. 2.29(a)) which has the same width irrespective of the time instance (it is based only on the stationary distribution of the wind power error). Therefore, since the wind power inaccuracy is increasing with time while the robustness region remains constant, we have the increasing pattern of Fig. 2.31. On the other hand, since the scenario approach (Fig. 2.30) is based on sampling, the span of the scenarios (“green” trajectories in Fig. 2.29(a)) does not exhibit this regularity pattern and covers a wider range of wind power values.

**Simulation conclusions**

The main conclusions drawn from our simulation results are summarized in Table I. As already mentioned, the probability of power deficit or surplus depends only on the approach used to solve the chance constrained problem and not on method 1 or 2, thus justifying the same characterization in the corresponding entries of Table I. The same holds for the conservatism of each method. By inspection of Fig. 6 and 7 it can be observed that the probability of power deficit or surplus when the scenario approach is used is lower compared to the one obtained by the quantile based approach, while both methods are below the design value of $\epsilon$. This implies that the scenario approach leads to a more conservative performance compared to the quantile based approach. This is justified by the fact that the scenario approach is based on sampling, so outliers may appear in the optimizations process, and by the fact that the bound which determines the number of scenarios $\tilde{N}$ is not tight. To rank the methods in terms of cost we use a numbering scheme where “1” implies low and “4” high cost. Independently of the approach used to solve the chance constraint, method 2 leads to lower cost compared to method 1 since it is shown in [103] that it constitutes a convex reformulation of the initial problem. Using the quantile based approach results in scheduling less reserves, thus leading to a lower scheduled cost.

Even though it seems more conservative in some cases, the scenario approach provides a more general framework to handle uncertainty since it takes into account the temporal correlation of the wind power error. This is of major importance especially if the optimization stages are coupled, as in the case where ramping constraints of the
generating units are taken into account. Moreover, subsequent developments of the sce-
nario approach [31], which are not exploited in this work, provide a way to reduce the
conservatism while providing the same performance guarantees.
Figure 2.29: (a) Wind power for one day of the simulated data. Forecast (“blue”), actual (“red”), scenarios used for methods 1 and 2 (“green”), and the span of the 10,000 wind power realizations used for evaluation purposes (shaded region). (b) Total scheduled cost (production + reserves) for method 1 (“solid” line) and method 2 (“dashed” line). The “blue” and “light blue” curves correspond to the scenario approach, whereas the “red” and “yellow” to the quantile based approach. (c) Power deficit for method 1 (“solid” line) and method 2 (“dashed” line). The “blue” and “light blue” curves correspond to the scenario approach, whereas the “red” and “yellow” to the quantile based approach.
Figure 2.30: Power deficit and surplus using the scenario approach, for one day of the simulated data, evaluated with 10,000 wind power realizations ($\epsilon = 10^{-1}$ and $\beta = 10^{-4}$). (a) Power surplus for method 2. (b) Probability of power surplus. (c) Power deficit for method 2. (d) Probability of power deficit.
Figure 2.31: Power deficit and surplus using the quantile based approach, for one day of the simulated data, evaluated with 10,000 wind power realizations ($\epsilon = 10^{-1}$ and $\beta = 10^{-4}$). (a) Power surplus for method 2. (b) Probability of wind power surplus. (c) Power deficit for method 2. (d) Probability of power deficit.
3 Conclusion

This document is the final report on modeling, analysis, impact and potential exploitation of all the methods developed within the MoVeS project where the three selected case studies defined the application context for this final assessment. It can be summarized as follows:

- **Microgrid energy management** case study validated the Model Checking method and its potential use for improved design of microgrids, and then also the Approximate Dynamic Programming, which helps speed-up the numerical solution of complex operational optimization problems. These results might be leveraged by companies deploying solutions for microgrids, which is a growing market that will pass $40 milliards in annual revenue globally by 2020.

- **Demand response** study was used to test the methods of Composition and Abstraction as well as Satisfiability Modulo Theory to reliably model behavior of large aggregations of thermostatically controlled loads. This is an important step enabling more efficient control of the residential loads in situations when the electricity supply and demand is not in balance. Applications of demand response represent a large business opportunity which is going to attract up to $3,000 millions of investment by the end of 2013. Although Europe is not the main demand response market, it is forecasted the applications will steadily grow by 2020.

- **N-1 security and reserve market** study analyzed the methods of Adaptive Dynamic Programming and Stochastic MPC, which were applied in a scenario that might significantly improve the overall process and software tools used by the Transmission System Operators for daily reserve scheduling.
Bibliography


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